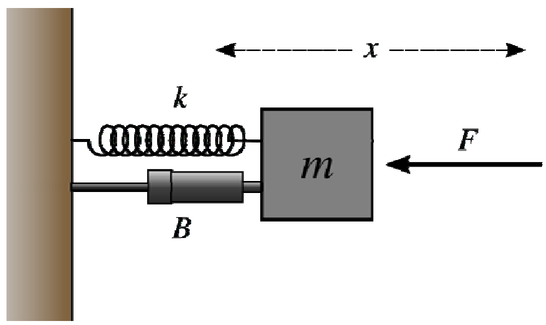
Nonlinear dynamics of a mass-spring-damper system

Background

Spring-mass systems are well-known in studies of mechanical vibrations (see sections 3.7 and 3.8 of the textbook).

The Model

In the present we study the dynamics of a mechanical system consisting of a block with a spring and a nonlinear damper (see the following figure courtesy of Wikipedia).



The model is formulated with

$$x'' + h(x, x') = F \cos \omega . t ,$$

where $h(x, x')$ is a function related to the spring and the nonlinear damper, $F \cos \omega t$ is a harmonic forcing term with amplitude F and circular frequency ω.

Objectives

We would like to (1) study dynamics of the model for different choices of function $h(x, x')$; (2) investigate the impacts of a nonlinear damper on the solution curves; (3) provide numerical simulations of the model; (4) provide physical interpretations of the solutions.

Project procedure

1. Let F = 0 and $h(x, x') = f(x, x') + g(x)$.The total energy of the system is given by $E = \frac{1}{2}(x')^2 + \int g(x)dx$. Show that $\frac{dE}{dt} = -x' f(x, x')$.

2. Let R be a region of phase plane (x, x') containing an equilibrium solution. Predict the behavior of solution curves in the region R for the cases (1) $\frac{dE}{dt} > 0$ and (2) $\frac{dE}{dt} < 0$ for all t.

3. Let F = 0 and $h(x, x') = |x'|x' + x$. Do the solution curves converge to the equilibrium solution $(x, x') = (0,0)$ or do they move away from it? Hint: use part (2).

4. Obtain the topographic curves to determine the stability of $(x, x') = (0,0)$. Procedure: Multiply the model by x', Let $y = x'$ and rewrite the model in the form of x and y. Take the term $|y|y^2$ to the right hand side of the equation and take the integral from both sides with respect to t from t_A to t_B, where $t_A < t_B$ Evaluate the left hand side integral and deduce that $\frac{1}{2}y^2 + \frac{1}{2}x^2$ constantly diminishes along every phase path passing though an arbitrary point A at time t_A and arrives at a point B at time t_B.

5. Let $y = x'$ and transform the model into a system of first order differential equations. Find the equilibrium solutions if any.

6. Use Matlab to plot the phase plane (x, y) corresponding to the solutions. You need to use pplan8.m.

7. Repeat parts (3), (5) and (6) for F = 0 and $h(x, x') = (x^2 + (x')^2 - 1)x' + x$. Describe your observations and numerically determine the stability of the limit cycle (See section 9.7 of the textbook to learn about the limit cycles).

8. Let $h(x, x') = (x^2 + (x')^2 - 1)x' + x$. Describe the changes in the phase plane as the value of F increases from zero. What are the effects of circular frequency ω on the phase plane?

9. Let $h(x, x') = (x^2 + (x')^2 - 1)x' + x$ and $y = x'$. Transform the model into a system of first order differential equations. Use Matlab to solve the model for different initial values. Note: Codes sirmodel.m SIRsolver.m available at the course webpage could be modified to generate the codes needed for solving the IVP. Also check the useful links toward the end of this document.

10. Provide physical interpretations of the solutions obtained in part (9)

11. A short essay (less than 150 words) of the main outcomes should be included in the supplementary documents.

Collegiality and Group Work

The groups in this class are meant to imitate real-world research groups. Each group should meet at least once a week. Each group member should (1) maintain a friendly environment for the entire group; (2) facilitate collaboration and problem solving; (3) provide a vision of the main objectives and ensure discussions lead to conclusions and decisions; (4) motivate and inspire other group members; (5) contribute to the group by sharing his/her knowledge, expertise and viewpoints; (6) participate in all meetings and discussions; (7) have productive suggestions to enhance quality of the work.

Instructions for presentation

(a) Each team is required to prepare around 15 PowerPoint slides.

(b) The contents of the presentation may include Introduction, Objectives, the Model, Analysis of the Model, Concluding Remarks, Limitations and Future Work.

(c) The first slide must contain the name and UIN of each participant. Also include the specific works (e.g essay writing, coding, analytical solutions) done by each participant.

(d) All figures must have labels and captions.

(e) Supplementary slides (e.g. Matlab codes, essay, further explanations, references) should be included in the same file after the presentation slides.

(f) Important note: please save your presentation in 97-2003 formats. There are compatibility issues with newer formats.

(g) Please send your presentation to mbani@math.tamu.edu before midnight Nov. 23, 2011. Also a print of the file should be given to me on Nov. 29.

Useful links

http://www.math.tamu.edu/REU/comp/matode.pdf (How to solve ODEs with Matlab)
http://www.mathworks.com/help/techdoc/ref/ode23.html (Matlab help for ODE45)