

Matlab for solving differential equations

To solve differential equations with Matlab, we may use the command “dsolve” in the following format:

dsolve('the differential equation', 'the initial condition, if any', 'the variable of differential equation')

Example 1: Find the solution of the following initial value problem

$y^{(4)}(x) - 3y(x) = 0$, with the initial condition $y(0) = 4$.

Enter dsolve('D4y - 3*y = 0', 'y(0)=4', 'x') in the command window. Then we get the solution

$C1 * \exp(-3^{1/4} * x) + (-C1 - C4 + 4) * \exp(3^{1/4} * x) - C3 * \sin(3^{1/4} * x) + C4 * \cos(3^{1/4} * x)$

Notes:

1. In the above solution, $C1, \dots, C4$ are arbitrary constants.
2. If dsolve cannot find an analytic solution for an equation, it prints the warning “Warning: explicit solution could not be found” and return an empty sym object.
3. There is no need to rewrite a differential equation based on $y(t)$ or $y(x)$. In the following example we find the solution $p(s)$ of a differential equation.

Example 2: Find the general solution of $p''(s) + e^{p(s)} = 0$

Entering dsolve('D2p+exp(p) = 0', 's'), gives

$\log(-1/2 * (-1 + \tanh(1/2 * (s + C2) / C1) ^ 2) / C1 ^ 2)$

(note that log in Matlab represents the natural log)

Example 3: Find the general solution of $3y''(t) - y'(t) + y = 0$

Enter dsolve('3*D2y - Dy + y = 0', 't') in the command window. We get

$C1 * \exp(1/6 * t) * \sin(1/6 * 11^{1/2} * t) + C2 * \exp(1/6 * t) * \cos(1/6 * 11^{1/2} * t)$

It is also possible to get the general solution of a differential equation with unknown coefficients.

Example 3: Find the general solution of $-3t y''(t) + ky'(t) = 0$, where k is a constant.

Enter dsolve('(-3*t)*D2y+k*y=0', 't') to get the general solution

$C1 * t^{1/2} * \text{besselj}(1, 2/3 * (-3 * k)^{1/2} * t^{1/2}) + C2 * t^{1/2} * \text{bessely}(1, 2/3 * (-3 * k)^{1/2} * t^{1/2})$

Where `besselj` and `bessely` are Bessel functions of first and second kinds, respectively.