A Numerical Model of Commodity Moisture Loss and Temperature Distribution During Refrigerated Storage

B.R. Becker and B.A. Fricke
Mechanical Engineering Department
University of Missouri-Kansas City
5100 Rockhill Road, Kansas City, MO 64110-2499 U.S.A.

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Abstract
A numerical model was developed to estimate the latent and sensible heat loads, moisture loss, and temperature distribution during the bulk storage of a wide variety of fruits and vegetables. A porous media approach was utilized to model the combined phenomena of transpiration, respiration, air flow, and convective heat and mass transfer. This paper discusses the modeling methodology utilized in the current computer algorithm and describes the development of the heat and mass transfer models. The results of the computer algorithm are compared to experimental data taken from the literature.

INTRODUCTION
This paper describes a numerical model which was developed to aid in the design of bulk refrigeration facilities for fruits and vegetables. A porous media model incorporating the combined phenomena of transpiration, respiration, air flow, and convective heat and mass transfer is used to estimate the latent and sensible heat loads during refrigerated storage of fruits and vegetables. The model also predicts the commodity moisture loss and temperature distribution.

A literature review revealed several existing heat transfer models for bulk commodity refrigeration (Bakker-Arkema and Bickert, 1966; Baird and Gaffney, 1976; Adre and Hellickson, 1989; Gan and Woods, 1989; Talbot et al., 1990; MacKinnon and Bilanski, 1992). However, these models do not adequately address the effects of transpiration, respiration, evaporative cooling and internal commodity temperature gradient. In addition, these models are commodity specific and do not estimate sensible and latent heat loads. Thus, the current numerical model was developed to estimate the latent and sensible heat loads as well as the moisture loss and temperature distribution in the bulk refrigeration of a wide variety of fruits and vegetables.

MODELING METHODOLOGY
As depicted in Figure 1, the computational model is based upon a one dimensional air flow pattern within a bulk load of fruits or vegetables. In the computational model, the bulk load is represented as a porous medium composed of "commodity computational cells." The conditioned air is modeled as "air parcels" which move through the "commodity computational cells."

Calculation commences with a specified initial temperature and humidity for the bulk load and the air contained within it. As shown in Figure 1a, the time-stepping begins with the first refrigerated "air parcel" moving into the first "commodity computational cell." At the same time, each of the initial "air parcels" moves from its original cell into the adjacent cell, while the "air parcel" within the last "commodity computational cell" moves from the bulk load into the plenum of the refrigeration unit. Within each "commodity computational cell," the transpiration rate, \( \dot{m} \), is calculated for the time-step,
The mass fraction of water vapor in each "air parcel" is then updated to reflect the effects of transpiration. Subsequently, within each cell, the heat generation due to respiration, $W$, the heat transfer from the commodity and the evaporative cooling due to transpiration are calculated for the time-step. Then, within each cell, the commodity temperature and the "air parcel" temperature are both updated to reflect the effects of respiration, heat transfer and evaporative cooling, thus completing the calculations for this time-step.

As shown in Figure 1b, the first "air parcel" moves to the second "commodity computational cell" and a newly conditioned second "air parcel" moves into the first "commodity computational cell." This second "air parcel" encounters the previously updated commodity temperature in the first "commodity computational cell."

As the time-stepping continues, each "air parcel" traverses the entire bulk load. The mass fraction of water vapor contained in each "air parcel," when it exits the bulk load, is used to calculate the latent heat load corresponding to that "air parcel," while its temperature is used to calculate its sensible heat load. As this algorithm time-steps towards a steady state, an estimate of the time histories of the latent and sensible heat loads, as well as moisture loss and temperature distribution, are obtained.

Figure 1. Computational model of refrigerated air flow through bulk load of commodity.

MASS TRANSFER CALCULATION

Transpiration is the moisture loss process composed of moisture transport through the commodity skin, evaporation from the commodity surface and convective mass transport to the surroundings. The driving force for transpiration is the water vapor pressure deficit between the commodity surface and the surrounding air:

$$\dot{m} = k_t (P_s - P_a)$$

The water vapor pressure at the commodity surface, $P_s$, is the saturation pressure at the commodity surface temperature. The water vapor pressure in the air, $P_a$, is a function of the mass fraction of water vapor in the air. Both $P_s$ and $P_a$ are evaluated at the previous time step by utilizing psychrometric relationships (ASHRAE, 1970).

Fockens and Meffert (1972) suggest that the transpiration coefficient, $k_t$, can be modeled as follows:
\[ k_i = \left[ \frac{1}{k_a} + \frac{1}{k_s} \right]^{-1} \]  

(2)

The air film mass transfer coefficient, \( k_a \), describes the convective mass transfer which occurs at the surface of the commodity. The air film mass transfer coefficient can be obtained from the Sherwood number, \( Sh \), via the following Sherwood-Reynolds-Schmidt correlation for convective mass transfer from a sphere (Geankoplis, 1978):

\[ Sh = \frac{k_a d}{\delta} = 2.0 + 0.552Re^{0.5}Sc^{0.33} \]  

(3)

The skin mass transfer coefficient, \( k_s \), describes the skin's diffusional resistance to moisture migration and is dependent upon the fraction of the commodity surface covered by pores. As such, it is theoretically difficult to determine the skin mass transfer coefficient, and thus, \( k_s \) must be determined experimentally (Chau et al., 1987; Gan and Woods, 1989).

During the time step, \( \Delta t \), the mass of water vapor in the air of the computational cell increases as follows:

\[ m_{H2O}^{i+1} = m_{H2O}^{0} + \dot{m}_s \Delta t \]  

(4)

The updated mass fraction of water vapor in the air of the computational cell, \( m_f^i \), then becomes:

\[ m_f^i = \frac{m_{H2O}^{i+1}}{m_{H2O}^{0} + \dot{m}_s \Delta t} \]  

(5)

This completes the transpiration calculations for one computational cell for the current time step.

**HEAT TRANSFER CALCULATION**

The bulk load was modeled as a porous medium composed of spherical commodities with uniform respiratory heat generation determined as a function of temperature via the Becker-Fricke correlations (Becker and Fricke, 1996). Furthermore, the temperature within a commodity was assumed to vary only in the radial direction. Thus, the governing form of the transient heat equation becomes:

\[ \frac{k}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \rho W = \rho c \frac{\partial T}{\partial t} \]  

(6)

An explicit finite difference technique was applied to Equation (6) by dividing a commodity into \( N \) spherical shells. The resulting finite difference equation applicable to the center node is given as follows:

\[ \frac{kA_i}{\Delta r} (T^0_i - T^0_0) + \rho v_i W_i = \frac{\rho c v_i (T^i_1 - T^0_0)}{\Delta t} \]  

(7)

The resulting finite difference equation applicable to the interior nodes is given as follows:

\[ \frac{kA_{i-1}}{\Delta r} (T^0_{i-1} - T^0_i) + \frac{kA_i}{\Delta r} (T^0_i - T^0_0) + \rho v_i W_i = \frac{\rho c v_i (T^i_1 - T^0_0)}{\Delta t} \]  

(8)

At the surface of the commodity, convection heat transfer, radiation heat transfer, and evaporative cooling due to transpiration must be considered. Thus, the finite difference equation at the surface becomes:
The effective heat transfer coefficient, $h_{\text{eff}}$, includes both convection and radiation:

$$h_{\text{eff}} = h_{\text{convection}} + h_{\text{radiation}}$$  \hspace{1cm} (10)

The convection heat transfer coefficient, $h_{\text{convection}}$, can be obtained from the Nusselt number, $Nu$, via the following Nusselt-Reynolds-Prandtl correlation (Geankoplis, 1978):

$$Nu = \frac{k_{\text{convection}} d}{k_{\text{air}}} = 2.0 + 0.552 Re^{0.53} Pr^{0.33}$$  \hspace{1cm} (11)

The radiation heat transfer coefficient, $h_{\text{radiation}}$, in Equation (10) is given by:

$$h_{\text{radiation}} = \sigma (T_r^4 + T_i^4) (T_r^4 + T_i^4)$$  \hspace{1cm} (12)

The formulation given by Equations (7), (8) and (9) defines the temperature distribution within a single commodity. However, Equation (9) requires knowledge of the temperature of the air parcel resident within the "commodity computational cell," $T_a^0$. This air temperature is determined at each time step by performing an energy balance between the air parcel and that portion of the bulk load which is contained within the "commodity computational cell:

$$n_i h_{\text{eff}} A_s (T_a^0 - T_N^0) = m_a^0 c_{p,a} \frac{(T_a^1 - T_a^0)}{\Delta t}$$  \hspace{1cm} (13)

Since Equations (7), (8), (9) and (13) are explicit finite difference equations, they can be solved directly for the updated nodal temperatures. The heat transfer calculation begins at the center node of the commodity and proceeds outward to the air parcel. This completes the heat transfer calculation for one computational cell for the current time step.

**EXPERIMENTAL VERIFICATION OF THE NUMERICAL MODEL**

To verify the numerical model’s accuracy, its calculated results were compared with experimental data on the bulk refrigeration of fruits and vegetables, obtained from the literature. Baird and Gaffney (1976) reported experimental data taken from bulk loads of oranges. They recorded commodity center and surface temperatures at the air exit of a bulk load for a period of two hours. The bulk load of oranges was 0.67 m deep and the commodities were initially at 32°C. The refrigerated air was at a temperature of -1.1°C with a velocity of 0.91 m/s. Figure 2 shows Baird and Gaffney's experimental data along with the output from the computer algorithm. Comparison of the model results with Baird and Gaffney’s data on oranges shows that the algorithm correctly predicts the trends of commodity temperatures with a maximum error of 1.4°C.

Brusewitz et al. (1992) conducted
experiments to determine moisture loss from peaches during post-harvest cooling. The post-harvest cooling was performed at 4°C, 92% relative humidity in a chamber with 20 air changes per minute for a period of four days. Peaches were picked in the morning when the ambient temperature was 16°C. Experimental data from Brusewitz et al. shows that the peaches lost 2.5% of their weight due to moisture loss during the four day cooling period. The numerical model predicted a weight loss of 2.53% at the end of the four day period, in good agreement with the experimental data. Figure 3 shows the results from the current computer algorithm as well as the experimental data.

CONCLUSIONS
This paper has described the development and performance of a numerical model which estimates the latent and sensible heat loads as well as the moisture loss and temperature distribution within a bulk load of fruits or vegetables. In the computational model, the bulk load is represented as a porous medium composed of "commodity computational cells" and the conditioned air is modeled as "air parcels" which move through these "commodity computational cells." A mass transfer model was developed to update the mass fraction of water vapor within each "commodity computational cell" at each time step. An explicit finite difference formulation of the transient heat equation in spherical coordinates was derived which accounts for both radiation and convection heat transfer at the commodity surface. This formulation yields the temperature distribution within the commodities resident in each "commodity computational cell" at each time step. It also yields the temperature of the "air parcel" resident within each "commodity computational cell" at each time step.

To verify the accuracy of the computer algorithm, its calculated results were compared with experimental data obtained from the literature. The results of these comparisons show good agreement between the numerical results and the experimental data for both temperature and moisture loss.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>( k_{\text{air}} )</th>
<th>( k_s )</th>
<th>( k_t )</th>
<th>( L )</th>
<th>( m_{H_2O}^0 )</th>
<th>( m_{H_2O}^1 )</th>
<th>( \dot{m} )</th>
<th>( \dot{m}_\text{t} )</th>
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<tr>
<td>( A_i )</td>
<td>surface area of ( i )th node</td>
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<td>( c )</td>
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<td>diameter of commodity</td>
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cell

$n_c$ number of commodities in computational cell

$N$ number of nodes

$Nu$ Nusselt number

$P_a$ ambient water vapor pressure

$P_s$ water vapor pressure at evaporating surface of commodity

$Pr$ Prandtl number

$r$ commodity radius

$Re$ Reynolds number

$Sc$ Schmidt number

$Sh$ Sherwood number

$t$ time

$T$ commodity temperature

$T_{a}$ dry bulb air temperature

$T_{a}^0$ air temperature at time $t$

$T_{a}^1$ air temperature at time $t + \Delta t$

$T_{i}^0$ temperature of $i^{th}$ node at time $t$

$T_{i}^1$ temperature of $i^{th}$ node at time $t + \Delta t$

$T_N^0$ temperature of surface node at time $t$

$T_N^1$ temperature of surface node at time $t + \Delta t$

$T_s$ product surface temperature

$T_{1}^0$ temperature of center node at time $t$

$T_{1}^1$ temperature of center node at time $t + \Delta t$

$v_i$ volume of $i^{th}$ node

$v_N$ volume of surface node

$W$ rate of heat generation of commodity per unit mass of commodity

$W_i$ rate of heat generation of commodity for node $i$

$W_N$ rate of heat generation of commodity for surface node

$W_1$ rate of heat generation of commodity for center node

$d$ coefficient of diffusion of water vapor in air

$\rho$ density of commodity

$s$ Stefan-Boltzmann constant

Literature Cited

ASHRAE. 1970. Brochure on Psychrometry. ASHRAE, Atlanta, GA.


