SOIL THERMAL CONDUCTIVITY:

EFFECTS OF SATURATION AND DRY DENSITY

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ABSTRACT

Saturation and dry density are important parameters governing soil thermal conductivity. An increase in either saturation or dry density of a soil will result in an increase in its conductivity. Both of these parameters should be accounted for in conductivity prediction methods. In this paper, empirical correlations are presented that show the dependence of thermal conductivity upon saturation. Also, a theoretical model is presented that gives an explicit dependence of conductivity upon dry density. In addition, this paper presents a review of existing prediction methods as well as a description of a data base of soil thermal conductivity measurements.

INTRODUCTION

A soil's thermal conductivity is significantly influenced by its saturation and dry density. Saturation describes the amount of moisture contained in a soil, while dry density refers to the mass of soil particles per unit volume. An increase in either the saturation or dry density of a soil will result in an increase in its thermal conductivity. Other factors that have a secondary effect upon soil thermal conductivity include mineral composition, temperature, texture, and time (Kersten 1949; Penner et al. 1975; Salomone et al. 1984; Salomone and Kovacs 1984; Salomone and Marlow 1989; Brandon and Mitchell 1989; Mitchell 1991; Becker et al. 1992). The purpose of this paper is to investigate the influences of saturation and dry density. Both of these parameters should be accounted for in soil thermal conductivity prediction algorithms.

This paper presents soil thermal conductivity correlations that were developed from measured data available in the literature. Due to the great impact that soil moisture content has on thermal conductivity, these correlations focus upon conductivity as a function of saturation. The correlations were developed for five soil types, namely, gravels, sands, silts, clays, and peats, in both the frozen and unfrozen states. These soil types correspond to those used in the Unified Soil Classification System (USCS). More exact soil classification into subclasses requires detailed knowledge of the soil's grain size distribution and its Atterberg Limits, namely, its liquid limit and its plastic limit. Since this detailed information is not generally available in the literature, the present paper considers only the loosely defined USCS classification.

The effect of soil dry density on thermal conductivity is studied by means of a mathematical model that was developed for predicting conductivity at low saturations. This model is based upon a microstructural approach and experimental data. Predictions generated by this model are compared with experimental results for low saturation levels in sandy soils.

LITERATURE SURVEY

Several soil thermal conductivity prediction methods exist in the literature. These include Van Rooyen and Winterkorn (1957), Johansen (1975), De Vries (1952), Gemant (1952), and Kersten (1949). These methods vary in applicability and complexity. A brief survey of these various methods is given below.

Van Rooyen and Winterkorn's (1957) correlation, based on data collected from sands and gravels, is given as follows:

$$\frac{1}{k} = A\,10^{-BS_r} + s \tag{1}$$

where k = soil thermal conductivity;

S_r = degree of saturation; and,

A, B, s = functions of dry density, mineral type, and granulometry, respectively.

The Van Rooyen-Winterkorn method is limited to unfrozen sands and gravels with saturation levels between 1.5% and 10%.

Johansen's (1975) correlation, which is based on thermal conductivity data for dry and saturated states at the same dry density, has the following form:

$$k = (k_{SAT} - k_D) \bullet k_e + k_D \qquad (2)$$

where k = soil thermal conductivity;

k_{SAT}, k_D = soil thermal conductivity in the saturated and dry states, respectively; and

k_e = a dimensionless function of soil saturation.

Johansen's method is suitable for calculating soil thermal conductivity of both coarse- and fine-grained soils in the frozen and unfrozen states. However, it is limited to saturations greater than 20%.

The correlation given by De Vries (1952) assumes that soil is a two-phase material composed of uniform ellipsoidal particles dispersed in a fluid phase. The De Vries correlation is given as

$$k = \frac{x_f\, k_f + F\, x_s\, k_s}{x_f + F\, x_s} \qquad (3)$$

where f, s = fluid and solid phases, respectively;

x = volume fraction; and

k = soil thermal conductivity.

The factor F is given by

$$F = \frac{1}{3} \sum_i \left[1 + \left(\frac{k_s}{k_f} - 1 \right) g_i \right]^{-1}, \quad i = a, b, c. \qquad (4)$$

In Equation 4, the g values, which sum to unity, were originally intended to be shape factors, but are usually used to fit empirical data. De Vries' method is applicable to unfrozen coarse soils with saturations between 10% and 20%.

Gemant's (1952) correlation is based upon an idealized geometrical model of soil particles with point contacts as depicted in Figure 1. Water is assumed to collect around the contact points to form a thermal bridge with heat flow assumed to be vertically upward. Gemant's correlation is given as follows:

$$\frac{1}{k} = \frac{[(1 - a)/a]^{4/3} \arctan[(k_s - k_w)/k_w]^{1/2}}{(h/2)^{1/3}[k_w(k_s - k_w)]^{1/2}} + \frac{(1 - z)}{k_s a} f\left\{\frac{b^2}{a}\right\} \tag{5a}$$

$$a = 0.078\, s^{1/2} \tag{5b}$$

$$h = 0.16 \times 10^{-3}\ sw - h_0 \tag{5c}$$

$$z = \left(\frac{1 - a}{a}\right)^{2/3} \left(\frac{h}{2}\right)^{1/3} \tag{5d}$$

$$b^2 = \left(\frac{a}{1 - a}\right)^{2/3} \left(\frac{h}{2}\right)^{2/3} \tag{5e}$$

In Equation 5, s is soil dry density, w is moisture content, h is the apex water (water collected around the contact points), h_0 is water absorbed as a film around the soil particles, k_s is the thermal conductivity of the solids, and k_w is the thermal conductivity of water. Gemant's method gives reasonable results for unfrozen sandy soils only.

Kersten (1949) tested many soil types and based his correlations on the empirical data he collected. He produced equations for frozen and unfrozen silt-clay soils and sandy soils. Kersten's correlations for unfrozen and frozen silt-clay soils are as follows:

$$\text{Unfrozen}: \; k = [0.9 \log w - 0.2] 10^{0.01\gamma_d}$$

$$\text{Frozen}: \; k = 0.01 (10)^{0.022\gamma_d} + 0.085 (10)^{0.008\gamma_d} w . \tag{6a}$$

(6b)

The correlations for sandy soils are as follows:

$$\text{Unfrozen}: \; k = [0.7 \log w + 0.4] 10^{0.01\gamma_d}$$

$$\text{Frozen}: \; k = 0.076 (10)^{0.013\gamma_d} + 0.032 (10)^{0.0146\gamma_d} \; w . \tag{7a}$$

(7b)

In Equations 6 and 7, k is soil thermal conductivity (Btu·in./ft^2·h·°F), w is moisture content, and $?_d$ is dry density. Kersten's correlations give reasonable results only for frozen soils with saturations up to 90%.

Farouki (1986) has studied the applicability of these methods and has suggested the conditions under which each method should be used. It is clear that these methods are applicable only for limited soil types and conditions, as shown in Table 1. Hence, they do not offer a unified methodology for the estimation of soil thermal conductivity applicable to a wide range of soil types and conditions. Therefore, these existing methods cannot be incorporated into numerical heat transfer algorithms.

In contrast, the correlations developed in this paper provide a unified methodology for evaluating soil thermal conductivity. These correlations are applicable to soils in five textural classes, namely, gravels, sands, silts, clays, and peats, in both the frozen and unfrozen states. Due to their unified format, these new correlations can be readily incorporated into numerical heat transfer algorithms.

DEVELOPMENT OF DATA BASE

In order to develop empirical correlations for soil thermal conductivity, a data base was created from measured data available in the literature. The measured soil thermal conductivity data reported in the literature were obtained by performing either a steady-state or a transient test.

In the steady-state method, a temperature gradient is applied to a soil sample until constant heat flow is obtained. Knowledge of the temperature gradient across the soil sample allows for the calculation of its thermal conductivity. Steady-state testing is time consuming and, because of this, the soil sample is susceptible to moisture diffusion. The resulting loss of moisture will affect the heat flow and thus the thermal conductivity (Kersten 1949; Penner et al. 1975; Farouki 1986). Of the data sources cited in this paper, only Kersten made use of the steady-state test.

The transient method involves inserting a thin, constant-flux heat probe into a soil sample. By knowing the heat flux and soil temperature history, the soil thermal conductivity can be calculated. Due to the shorter time requirement, moisture migration is decreased in the transient test as compared to the steady-state test. This usually results in a more accurate measurement of soil thermal conductivity (Penner et al. 1975; Salomone et al. 1984; Salomone and Kovacs 1984; Salomone and Marlow 1989; Farouki 1986).

In the work described in this paper, thermal conductivity data at various dry densities, moisture contents, and temperatures were collected for each soil type. To obtain reasonable results, many sources of data were consulted: Kersten (1949), Penner et al. (1975), Salomone and Marlowe (1989), De Vries (1952), Farouki (1986), Andersland and Anderson (1978), Nakshabandi and Kohnke (1965), and Sawada (1977). Based upon texture, the soil data were classified into five general types--gravel, sand, silt, clay, and peat. A brief description of each of the five soil samples that constitutes the data base is given below.

Gravel

Most of the measured data on gravels is from Kersten (1949). These data include Chena River gravel, which is mainly composed of quartz and igneous rock with sizes ranging from 0.10 to 0.75 in. (2.5 to 19 mm).

Sand

The measured data on sand were collected from the works of Kersten (1949), Salomone and Marlow (1989), De Vries (1952), Andersland and Anderson (1978), Nakshabandi and Kohnke (1965), and Sawada (1977).

Kersten presented data on 12 sand samples, of which five were natural sands and seven were man-made. The five natural sands include Fairbanks sand, Lowell sand, Northway sand, Northway fine sand, and Dakota sandy loam. The Fairbanks sand was a siliceous sand with 27.5% of the particles larger than 0.079 in. (2.0 mm) and 70% of the particles between 0.020 and 0.079 in. (0.5 and 2.0 mm). The Lowell sand was also siliceous, with particles between 0.02 and 0.079 in. (0.5 and 2.0 mm). The two Northway sands are similar in their composition, with their main constituent being feldspar with grain sizes ranging from 0.19 to 0.0030 in. (4.75 mm to 0.075 mm). No details are available on the Dakota sandy loam.

Of the seven man-made sands, three were feldspar sands and four were quartz sands. The feldspar sands consisted of 90% sand-sized particles and 10% gravel-sized particles. The quartz sands included one sample with grain sizes larger than 0.020 in. (0.5 mm) and three samples with grain sizes between 0.020 and 0.079 in. (0.5 mm to 2.0 mm).

The sands tested by Salomone and Marlow (1989) were classified according to the Unified Soil Classification System (USCS). These sands included well-graded sands (SW), poorly graded sands (SP), silty sands (SM), and clayey sands (SC). However, no information was available concerning their mineral constituents.

The remaining sands were fine-grained sands; however, no information is available on their grain size distributions or mineral constituents.

Silt

The measured data on silt are from Kersten (1949) and Salomone and Marlow (1989). Kersten tested three silts: Northway silt loam, Fairbanks silt loam, and Fairbanks silty clay loam. All three silts were classified as low-plasticity silts (ML) according to the USCS.

Salomone and Marlow presented data for several low plasticity silts. Little information is available on the mineral constituents of these silts.

Clay

The measured data on clay are from Kersten (1949), Salomone and Marlow (1989), and Penner et al. (1975). Kersten tested two clays--Ramsey sandy loam and Healy clay--both of which were classified as low-plasticity clays (CL). The main mineral constituent of these clays is kaolinite. Salomone and Marlow tested both high- and low-plasticity clays; however, no information was given concerning the mineral composition of these clays. The clay samples tested by Penner et al. were low-plasticity clays containing quartz, illite, chlorite, and kaolinite.

Peat

The measured data on peat are from Kersten (1949) and Salomone and Marlow (1989). Kersten tested Fairbanks peat while Salomone and Marlow tested highly decomposed woody peat.

EFFECTS OF SATURATION

Basic Definitions

An expression for saturation can be derived from the basic definitions of dry density, solid density, and moisture content. Dry density, ?$_d$, and solid density, ?$_s$, are defined as follows:

$$\rho_d = \frac{M_S}{V_T} \tag{8a}$$

$$\rho_S = \frac{M_S}{V_S} \tag{8b}$$

where M_s = mass of solid soil particles,

V_S = volume of the solid particles, and,

V_T = total volume.

Moisture content, w, and saturation, S, are given as follows:

$$w = \frac{M_W}{M_S} \tag{9a}$$

$$S = \frac{V_W}{V_V} \tag{9b}$$

where M_W = mass of water,

V_W = volume of water, and

V_V = volume of void spaces.

Combining Equations 8 and 9 yields the following expression for saturation, in which ?$_w$ is the density of water:

$$S = \frac{r_d w}{r_w \left(1 - \frac{r_d}{r_S}\right)} \_ \ 100\% . \tag{10}$$

Thermal Conductivity vs. Saturation

As depicted in Figure 2, the thermal conductivity of a soil increases in three stages as the saturation level increases. At low saturations, moisture first coats the soil particles. The gaps between the soil particles are not filled rapidly and thus there is a slow increase in thermal conductivity. When the particles are fully coated with moisture, a further increase in the moisture content fills the voids between particles. This increases the heat flow between particles, resulting in a rapid increase in thermal conductivity. Finally, when all the voids are filled, further increasing the moisture content no longer increases the heat flow, and the thermal conductivity does not appreciably increase. The model used to describe this behavior is as follows:

$$S = l_1 [\sinh(l_2 k + l_3) - \sinh(l_4)] \tag{11}$$

where S = saturation,

k	=	soil thermal conductivity (Btu·in./ft^2·h·°F), and
$?_1$, $?_2$, $?_3$, $?_4$	=	coefficients that depend upon soil type.

The values of $?_1$ through $?_4$ for each of the five soil types in both the frozen and unfrozen states are given in Table 2. At a saturation of zero, Equation 11 reduces to the following:

$$l_2 k_0 + l_3 = l_4 . \qquad (12)$$

Equation 12 shows that the coefficient $?_4$ is related to the thermal conductivity of dry soil, k_0. Figures 3 th 3 through 7. Three curves have been given for each soil type (except peat). The upper curve represents the upper limit of the measured data, the middle curve is the mean of the measured data, and the lower curve represents the lower limit of the measured data. Due to the small amount of measured data for peaty soils, only a mean correlation is presented. Measured data collected for gravel include saturations up to approximately 40% and, thus, the correlations for gravel are valid only to 40% saturation.

An error analysis of these correlations is presented in the work by Becker et al. (1992). The difference, Z, between the mean correlation and the measured data was calculated at each data point. A normalized difference, Z^*, was calculated as $Z^* = (Z -)/s_Z$, in which is the mean of the calculated differences and s_Z is the standard deviation of those differences. The cumulative frequency of the normalized difference, Z^*, was compared to a cumulative normal distribution function. This error analysis shows that these correlations provide a good fit to the measured data.

EFFECTS OF DRY DENSITY

At any given saturation level, the soil thermal conductivity exhibits considerable variation, as shown in Figures 3 through 7. This variation is due, in part, to differences in dry density. The effect of dry density upon soil thermal conductivity was studied by means of a mathematical model that was developed for a particulate system composed of random arrays of identical spheres in an almost dry state, as depicted in stages 1 and 2 of Figure 2. The heat transfer in this simple system can be idealized to occur in the neighborhood of the interparticle contacts. Based upon this model, under low confining stress, the expression for thermal conductivity as a function of dry density was found to be (Misra et al. 1992)

$$k = k_a \left(10.68 \frac{r_d}{r_s} - 4\right) \left(\ln a^2 + b + wc - 3.9\right). \tag{13}$$

Equation 13 shows that thermal conductivity varies linearly with dry density. The measured data exhibit a similar behavior. Also, Equation 13 accounts for the presence of moisture at low levels. This behavior is similar to the experimental results reported by Brandon and Mitchell (1989) for sands with low moisture content. In Figure 8, a comparison is presented between the predictions and the measurements for thermal conductivity vs. dry density for quartz sands at three levels of moisture content: w = 0.0%, 0.5%, and 1.0%. In these predictions, the thermal conductivity of a solid particle is taken to be 58.25 Btu·in./ft^2·h·°F (8.39 W/m·°C) and that of air is taken to be 0.18 Btu·in./ft^2·h·°F (0.026 W/m·°C). The parameter, b, is set to -3.1 and c is set to 10.5. As shown in Figure 8, a good agreement exists between the predictions and the measured data.

CONCLUSIONS

This paper has focused upon the influence that soil saturation and dry density have on thermal conductivity. To this end, a family of empirical correlations was presented that relates thermal conductivity to saturation. The effects of dry density were investigated by means of a microstructural, theoretical model. Taken together, the empirical correlations and the theoretical model correctly represent the influence of saturation and dry density upon soil thermal conductivity.

This paper also presented a review of existing prediction methods as well as a description of a data base of thermal conductivity measurements.

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Table 1.

Applicability of Prediction Methods[a]

State	Texture	Saturation	Method
Unfrozen	Coarse Grained	0.015 - 0.100	Van Rooyen and Winterkorn (except for low-quartz crushed rock)
		0.100 - 0.200	De Vries
		0.200 - 1.000	Johansen
		0.000 - 1.000	Gemant (sandy silt-clay)
		saturated	Johansen, De Vries, Gemant
	Fine Grained	0.000 - 0.100	Johansen (underpredicts by 15%)
		0.100 - 0.200	Johansen (underpredicts by 5%)
		0.200 - 1.000	Johansen
		saturated	Johansen, De Vries, Gemant
Frozen	Coarse Grained	0.100 - 1.000	Johansen
		saturated	Johansen, De Vries
	Fine Grained	0.000 - 0.900	Kersten
		0.100 - 1.000	Johansen
		saturated	Johansen, De Vries

[a]Data from Farouki (1986).

TABLE 2

Correlation Coefficients

Soil Type	Frozen / Unfrozen	?$_1$			?$_2$			?$_3$			?$_4$		
		Low	Mid	High	Low	Mid	High	Low	Mid	High	Low	Mid	High
Clay	Frozen	23.5	14.5	14.0	0.25	0.25	0.25	-2.0	-2.5	-3.0	-1.75	-2.0	-2.0
	Unfrozen	33.5	27.0	14.0	0.29	0.265	0.32	-1.6	-1.5	-3.0	-1.31	-0.97	-1.72
Gravel	Frozen	25.4	11.0	11.3	0.29	0.35	0.3	-2.1	-3.0	-2.8	-1.23	-1.6	-0.85
	Unfrozen	16.5	6.5	8.3	0.32	0.38	0.2	-1.9	-3.0	-1.8	-1.1	-1.48	-0.8
Peat	Frozen		12.0			0.4			-2.6			-2.52	
	Unfrozen		28.0			0.865			-1.9			-1.4675	
Sand	Frozen	26.0	10.0	15.0	0.265	0.24	0.17	-1.0	-2.2	-1.8	-0.735	-1.625	-0.44
	Unfrozen	6.4	6.8	6.8	0.8	0.4	0.5	-3.2	-2.9	-7.5	-2.0	-1.5	-2.0
Silt	Frozen	38.0	19.5	18.5	0.24	0.27	0.2	-1.2	-1.8	-2.0	-0.96	-1.53	-1.8
	Unfrozen	28.0	17.0	22.0	0.4	0.4	0.25	-1.0	-2.6	-2.2	-0.6	-1.6	-0.95