EFFECTS OF SATURATION AND DRY DENSITY

ON SOIL THERMAL CONDUCTIVITY

by

Bryan R. Becker, Ph.D., P.E.
Associate Professor

and

Brian A. Fricke
Research Assistant

Department of Mechanical and Aerospace Engineering
University of Missouri-Kansas City
5605 Troost Avenue
Kansas City, MO 64110-2823

1 August 1997

EFFECTS OF SATURATION AND DRY DENSITY ON SOIL THERMAL CONDUCTIVITY

Bryan R. Becker, Ph.D., P.E. and *Brian A. Fricke*
Department of Mechanical and Aerospace Engineering
University of Missouri-Kansas City
5605 Troost Avenue
Kansas City, MO 64110-2823
U.S.A.

KEY WORDS

Soil, thermal conductivity, saturation, dry density.

ABSTRACT

A soil's thermal conductivity is significantly influenced by its saturation and dry density. Saturation describes the amount of moisture contained in a soil, while dry density refers to the mass of soil particles per unit volume. An increase in either the saturation or dry density of a soil will result in an increase in its thermal conductivity. Other factors that have a secondary effect upon soil thermal conductivity include mineral composition, temperature, texture, and time [1-8]. The purpose of this paper is to investigate the influences of saturation and dry density. Both of these parameters should be accounted for in soil thermal conductivity prediction algorithms.

This paper presents soil thermal conductivity correlations that were developed from measured data available in the literature. Due to the great impact that soil moisture content has on thermal conductivity, these correlations focus upon conductivity as a function of saturation. The correlations were developed for five soil types, namely, gravels, sands, silts, clays, and peats, in both the frozen and unfrozen states. These soil types correspond to those used in the Unified Soil Classification System (USCS). More exact soil classification into subclasses requires detailed knowledge of the soil's grain size distribution and its Atterberg Limits, namely, its liquid limit and its plastic limit. Since this detailed information is not generally available in the literature, the present paper considers only the loosely defined USCS classification.

INTRODUCTION

Accurate estimates of a soil's thermal conductivity are of prime importance in the numerical simulation of heat transmission through soils. Soil thermal conductivity estimation methods that could easily be incorporated in computer algorithms will have wide application. They will facilitate the efficient design of ground source heat pump systems and will help ensure the safe design and location of underground storage facilities for nuclear and other hazardous waste materials. Furthermore, the resultant prediction methods will be useful in calculation of heat loss through basements, slabs and crawl spaces. The focus of this paper is to develop correlations of soil thermal conductivity which can be included in numerical heat transfer algorithms.

Factors affecting soil thermal conductivity include moisture content, dry density, mineral composition and temperature [1-5]. Moisture content, by far, has the greatest impact upon soil thermal conductivity. As moisture is added to a soil, a thin water film develops which bridges the gaps between the soil particles. This "bridging" increases the effective contact area between the soil particles, which increases the heat flow and results in higher thermal conductivity. As the voids between the soil particles become completely filled with moisture, the soil thermal conductivity no longer increases with increasing moisture content [3-5].

Soil thermal conductivity also increases with the dry density of the soil. With an increase in soil dry density, more soil particles are packed into a unit volume and the number of contact points between the particles increases. This increase in contact points provides a larger heat flow path and, thus, increases the soil thermal conductivity [3-5].

Soil thermal conductivity also varies with the mineral composition of the soil. For example, sands with a high quartz content generally have a greater thermal conductivity than sands with high contents of plagioclase feldspar and pyroxene [1].

Except at the ice point, soil thermal conductivity varies little with temperature. However, a dramatic change in soil thermal conductivity occurs between the frozen and unfrozen states due to the higher thermal conductivity of ice [1,2,5].

In this paper correlations are developed for soil thermal conductivity as a function of moisture content. Correlations are presented for various soil types, namely, gravels, sands, silts, clays and peats, in both the frozen and unfrozen states. The effect of dry density on a soil's thermal conductivity is also studied. The developed correlations are based upon measured thermal conductivity data which are available in the literature. The data includes laboratory as well as field measurements.

LITERATURE SURVEY

Several soil thermal conductivity prediction methods exist in the literature. These include Van Rooyen et al. [9], Johansen [10], De Vries [11], Gemant [12], and Kersten [1]. These methods vary in applicability and complexity. A brief survey of these various methods is given below.

The Van Rooyen et al. [9] correlation, based on data collected from sands and gravels, is given as follows:

$$\frac{1}{k} = A\,10^{-BS_r} + s \tag{1}$$

where k is the soil thermal conductivity, S_r is the degree of saturation, and A, B and s are functions of dry density, mineral type, and granulometry, respectively. The Van Rooyen method is limited to unfrozen sands and gravels with saturation levels between 1.5% and 10%.

Johansen's [10] correlation, which is based on thermal conductivity data for dry and saturated states at the same dry density, has the following form:

$$k = (k_{SAT} - k_D) \bullet k_e + k_D \tag{2}$$

where k is the soil thermal conductivity, k_{SAT} and k_D are the soil thermal conductivity in the saturated and dry states, respectively, and k_e is a dimensionless function of soil saturation. Johansen's method is suitable for calculating soil thermal conductivity of both coarse- and fine- grained soils in the frozen and unfrozen states. However, it is limited to saturations greater than 20%.

The correlation given by De Vries [11] assumes that soil is a two-phase material composed of uniform ellipsoidal particles dispersed in a fluid phase. The De Vries correlation is given as

$$k = \frac{x_f\,k_f + F\,x_s\,k_s}{x_f + F\,x_s} \tag{3}$$

where the subscripts *f* and *s* represent the fluid and solid phases, respectively, *x* is the volume fraction, and *k* is the soil thermal conductivity. The factor *F* is given by

$$F = \frac{1}{3} \sum_i \left[1 + \left(\frac{k_s}{k_f} - 1 \right) g_i \right]^{-1} , \ i = a , b , c . \tag{4}$$

In Equation (4), the *g* values, which sum to unity, were originally intended to be shape factors, but are usually used to fit empirical data. De Vries' method is applicable to unfrozen coarse soils with saturations between 10% and 20%.

Gemant's [12] correlation is based upon an idealized geometrical model of soil particles with point contacts as depicted in Figure 1. Water is assumed to collect around the contact points to form a thermal bridge with heat flow assumed to be vertically upward. Gemant's correlation is given as follows:

$$\frac{1}{k} = \frac{[(1 - a)/a]^{4/3} \arctan [(k_s - k_w)/k_w]^{1/2}}{(h/2)^{1/3} [k_w (k_s - k_w)]^{1/2}} + \frac{(1 - z)}{k_s a} f \left\{ \frac{b^2}{a} \right\} \tag{5a}$$

$$a = 0.078 \, s^{1/2} \tag{5b}$$

$$h = 0.16 \times 10^{-3} \, sw - h_0 \tag{5c}$$

$$z = \left(\frac{1 - a}{a} \right)^{2/3} \left(\frac{h}{2} \right)^{1/3} \tag{5d}$$

$$b^2 = \left(\frac{a}{1 - a} \right)^{2/3} \left(\frac{h}{2} \right)^{2/3} \tag{5e}$$

In Equation (5), *s* is the soil dry density, *w* the is moisture content, *h* is the apex water (water collected around the contact points), h_0 is the water absorbed as a film around the soil particles, k_s is the thermal conductivity of the solids, and k_w is the thermal conductivity of water. Gemant's method gives reasonable results for unfrozen sandy soils only.

Kersten [1] tested many soil types and based his correlations on the empirical data he collected. He produced equations for frozen and unfrozen silt-clay soils and sandy soils. Kersten's correlations for unfrozen and frozen silt-clay soils are as follows:

$$\text{Unfrozen}: \; k = [0.130 \log w \; - \; 0.0288]\,10^{0.000624\rho_d}$$

$$\text{Frozen}: \; k = 0.00144\,(10)^{0.00137\rho_d} + 0.0123\,(10)^{0.000499\rho_d}\,w$$

(6a)

(6b)

The correlations for sandy soils are as follows:

$$\text{Unfrozen}: \; k = [0.101 \log w \; + \; 0.0577]\,10^{0.000624\rho_d}$$

$$\text{Frozen}: \; k = 0.0110\,(10)^{0.000812\rho_d} + 0.00462\,(10)^{0.000911\rho_d}\,w$$

(7a)

(7b)

In Equations (6) and (7), k is the soil thermal conductivity (W/m·°C); w is moisture content; and ρ_d is the dry density (kg/m^3). The equations for the silt and clay soils apply for moisture contents of 7% or more; those for the sandy soils, of 1% or more. Kersten's correlations give reasonable results only for frozen soils with saturations up to 90%.

Farouki [13] has studied the applicability of these methods and has suggested the conditions under which each method should be used. It is clear that these methods are applicable only for limited soil types and conditions, as shown in Table 1. Hence, they do not offer a unified methodology for the estimation of soil thermal conductivity applicable to a wide range of soil types and conditions. Therefore, these existing methods cannot be incorporated into numerical heat transfer algorithms.

In contrast, the correlations developed in this paper provide a unified methodology for evaluating soil thermal conductivity. These correlations are applicable to soils in five textural classes, namely, gravels, sands, silts, clays, and peats, in both the frozen and unfrozen states. Due to their unified format, these new correlations can be readily incorporated into numerical heat transfer algorithms.

DEVELOPMENT OF DATA BASE

In order to develop empirical correlations for soil thermal conductivity, a data base was created from measured data available in the literature. The measured soil thermal conductivity data reported in the literature were obtained by performing either a steady-state or a transient test.

In the steady-state method, a temperature gradient is applied to a soil sample until constant heat flow is obtained. Knowledge of the temperature gradient across the soil sample allows for the calculation of its thermal conductivity. Steady-state testing is time consuming and, because of this, the soil sample is susceptible to moisture diffusion. The resulting loss of moisture will affect the heat flow and thus the thermal conductivity [1,2,13]. Of the data sources cited in this paper, only Kersten made use of the steady-state test.

The transient method involves inserting a thin, constant-flux heat probe into a soil sample. By knowing the heat flux and soil temperature history, the soil thermal conductivity can be calculated. Due to the shorter time requirement, moisture migration is decreased in the transient test as compared to the steady-state test. This usually results in a more accurate measurement of soil thermal conductivity [2-5,13].

In the work described in this paper, thermal conductivity data at various dry densities, moisture contents, and temperatures were collected for each soil type. To obtain reasonable results, many sources of data were consulted [1,2,5,11,13-16]. Based upon texture, the soil data were classified into five general types--gravel, sand, silt, clay, and peat. A brief description of each of the five soil samples that constitutes the data base is given below.

Gravel

Most of the measured data on gravels is from Kersten [1]. These data include Chena River gravel, which is mainly composed of quartz and igneous rock with sizes ranging from 2.5 to 19 mm.

Sand

The measured data on sand were collected from the works of Kersten [1], Salomone et al. [5], De Vries [11], Andersland et al. [14], Nakshabandi et al. [15], and Sawada [16].

Kersten presented data on 12 sand samples, of which five were natural sands and seven were man-made. The five natural sands include Fairbanks sand, Lowell sand, Northway sand, Northway fine sand, and Dakota sandy loam. The Fairbanks sand was a siliceous sand with 27.5% of the particles larger than 2.0 mm and 70% of the particles between 0.5 and 2.0 mm. The Lowell sand was also siliceous, with particles between 0.5 and 2.0 mm. The two Northway sands are similar in their composition, with their main constituent being feldspar with grain sizes ranging from 4.75 mm to 0.075 mm. No details are available on the Dakota sandy loam.

Of the seven man-made sands, three were feldspar sands and four were quartz sands. The feldspar sands consisted of 90% sand-sized particles and 10% gravel-sized particles. The quartz sands included one sample with grain sizes larger than 0.5 mm and three samples with grain sizes between 0.5 mm to 2.0 mm.

The sands tested by Salomone et al. [5] were classified according to the Unified Soil Classification System (USCS). These sands included well-graded sands (SW), poorly graded sands (SP), silty sands (SM), and clayey sands (SC). However, no information was available concerning their mineral constituents.

The remaining sands were fine-grained sands; however, no information is available on their grain size distributions or mineral constituents.

Silt

The measured data on silt are from Kersten [1] and Salomone et al. [5]. Kersten tested three silts: Northway silt loam, Fairbanks silt loam, and Fairbanks silty clay loam. All three silts were classified as low-plasticity silts (ML) according to the USCS. Salomone et al. presented data for several low plasticity silts. Little information is available on the mineral constituents of these silts.

Clay

The measured data on clay are from Kersten [1], Salomone et al. [5], and Penner et al. [2]. Kersten tested two clays--Ramsey sandy loam and Healy clay--both of which were classified as low-plasticity clays (CL). The main mineral constituent of these clays is kaolinite. Salomone et al. tested both high- and low-plasticity clays; however, no information was given concerning the mineral composition of these clays. The clay samples tested by Penner et al. were low-plasticity clays containing quartz, illite, chlorite, and kaolinite.

Peat

The measured data on peat are from Kersten [1] and Salomone et al. [5]. Kersten tested Fairbanks peat while Salomone and Marlow tested highly decomposed woody peat.

EFFECTS OF SATURATION

Basic Definitions

An expression for saturation can be derived from the basic definitions of dry density, solid density, and moisture content. Dry density, ρ_d, and solid density, ρ_s, are defined as follows:

$$\rho_d = \frac{M_S}{V_T} \tag{8a}$$

$$\rho_s = \frac{M_S}{V_S} \tag{8b}$$

where M_S is the mass of solid soil particles, V_S is the volume of the solid particles and V_T is the total volume. Moisture content, w, and saturation, S, are given as follows:

$$w = \frac{M_W}{M_S} \tag{9a}$$

$$S = \frac{V_W}{V_V} \tag{9b}$$

where M_W is the mass of water, V_W is the volume of water and V_V is the volume of void spaces. Combining Equations (8) and (9) yields the following expression for saturation, in which ρ_w is the density of water:

$$S = \frac{\rho_d w}{\rho_w \left(1 - \frac{\rho_d}{\rho_s}\right)} \times 100\% . \tag{10}$$

Thermal Conductivity vs. Saturation

As depicted in Figure 2, the thermal conductivity of a soil increases in three stages as the saturation level increases. At low saturations, moisture first coats the soil particles. The gaps between the soil particles are not filled rapidly and thus there is a slow increase in thermal conductivity. When the particles are fully coated with moisture, a further increase in the moisture content fills the voids between particles. This increases the heat flow between particles, resulting in a rapid increase in thermal conductivity. Finally, when all the voids are filled, further increasing the moisture content no longer increases the heat flow, and the thermal conductivity does not appreciably increase. The model used to describe this behavior is as follows:

$$S = \lambda_1 [\sinh(\lambda_2 k + \lambda_3) - \sinh(\lambda_4)] \tag{11}$$

where S is the saturation, k is the soil thermal conductivity (W/m·°C) and λ_1, λ_2, λ_3 and λ_4 are coefficients that depend upon soil type.

The values of λ_1 through λ_4 for each of the five soil types in both the frozen and unfrozen states are given in Table 2. At a saturation of zero, Equation 11 reduces to the following:

$$\lambda_2 k_0 + \lambda_3 = \lambda_4 . \tag{12}$$

Equation (12) shows that the coefficient $?_4$ is related to the thermal conductivity of dry soil, k_0. Figures 3 through 7 present the measured soil thermal conductivity versus saturation data for the five soil types in both the frozen and unfrozen states. The empirical correlations, based upon Equation (11), are also plotted in Figures 3 through 7. Three curves have been given for each soil type (except peat). The upper curve represents the upper limit of the measured data, the middle curve is the mean of the measured data, and the lower curve represents the lower limit of the measured data. Due to the small amount of measured data for peaty soils, only a mean correlation is presented. Measured data collected for gravel include saturations up to approximately 40% and, thus, the correlations for gravel are valid only to 40% saturation.

An error analysis of these correlations is presented in the work by Becker et al. [8]. The difference, Z, between the mean correlation and the measured data was calculated at each data point. A normalized difference, Z^*, was calculated as $Z^* = (Z -)/s_Z$, in which is the mean of the calculated differences and s_Z is the standard deviation of those differences. The cumulative frequency of the normalized difference, Z^*, was compared to a cumulative normal distribution function. This error analysis shows that these correlations provide a good fit to the measured data.

EFFECTS OF DRY DENSITY

At any given saturation level, the soil thermal conductivity exhibits considerable variation as shown in Figures 3 through 7. This variation is due, in part, to differences in dry density. Soil thermal conductivity increases with the dry density of the soil. As shown in Figure 8, the relationship between dry density and thermal conductivity is linear at a given level of saturation. As shown in Figure 9, when the dry density increases, the number of contact points between the soil particles increases [17]. This increase in contact points results in an increase in the effective heat flow area which causes an increase in soil thermal conductivity.

At a given dry density and saturation level, the scatter of soil thermal conductivity is due, in part, to the shape of the soil particles. Figure 9 shows that particle shape influences how well particles can be packed together. Soils with relatively flat surfaces have a larger number of contact points, thus resulting in greater thermal conductivity as compared to soils composed of round particles which have a smaller number of contact points.

CONCLUSIONS

In conclusion, a review of existing soil thermal conductivity prediction methods was presented and a soil thermal conductivity database was constructed from measured data available in the literature. From this database, a family of empirical correlations was developed which relate soil thermal conductivity to saturation for five soil types, namely, gravel, sand, silt, clay, and peat, in both the frozen and unfrozen states. The error analysis presented by Becker et al. [8] shows that these correlations provide a good fit to the measured data available in the literature. In addition, dry density was shown to significantly influence the scatter of soil thermal conductivity at a given saturation level.

REFERENCES

[1] Kersten, M.S. 1949. "Thermal properties of soils," Bulletin 28, Minneapolis: Engineering Experiment Station, University of Minnesota.

[2] Penner, E., et al. 1975. "Thermal conductivity laboratory studies of some MacKenzie highway soils," Canadian Geotechnical Journal, Vol. 12, No. 3, pp. 271-288.

[3] Salomone, L.A., et al. 1984. "Thermal performance of fine-grained soils," Journal of Geotechnical Engineering, Vol. 110, No. 3, pp. 359-374.

[4] Salomone, L.A., et al. 1984. "Thermal resistivity of soils," Journal of Geotechnical Engineering, Vol. 110, No. 3, pp. 375-389.

[5] Salomone, L.A., et al. 1989. "Soil rock classification according to thermal conductivity," EPRI CU-6482, Palo Alto, CA: Electric Power Research Institute.

[6] Brandon, T.L., et al. 1989. "Factors influencing thermal resistivity of sands," Journal of Geotechnical Engineering, Vol. 115, No. 12, pp. 1683-1698.

[7] Mitchell, J.K. 1991. "Conduction phenomena: From theory to geotechnical practice," Geotechnique, Vol. 41, No. 3, pp. 299-340.

[8] Becker, B.R., et al. 1992. "Development of correlations for soil thermal conductivity," International Communications in Heat and Mass Transfer, Vol. 19, pp. 59-68.

[9] Van Rooyen, M., et al. 1957. "Structural and textural influences on thermal conductivity of soils," Highway Research Board Proceedings, Vol. 39, pp. 576-621.

[10] Johansen, O. 1975. "Thermal conductivity of soils," Ph.D. thesis. Trondheim, Norway (CRREL Draft Translation 637, 1977) ADA 044002.

[11] De Vries, D.A. 1952. "The thermal conductivity of soil," Mededelingen van de Landbouwhogeschool te Wageningen, Vol. 52, No. 1, pp. 1-73.

[12] Gemant, A. 1952. "How to compute thermal soil conductivities," Heating, Piping, and Air Conditioning, Vol. 24, No. 1, pp. 122-123.

[13] Farouki, O.T. 1986. Thermal Properties of Soils, Clausthal-Zellerfeld, Germany: Trans Tech Publications.

[14] Andersland, O.B., et al. 1978. Geotechnical Engineering for Cold Regions. New York: McGraw Hill.

[15] Nakshabandi, G., et al. 1965. "Thermal conductivity and diffusivity of soils as related to moisture tension and other physical properties," Agricultural Meteorology, Vol. 2, pp. 271-279.

[16] Sawada, S. 1977. "Temperature dependence of thermal conductivity of frozen soil," Research Report, 9(1). Kirami, Hokkaido, Japan: Kirami Technical College.

[17] Chang, C.S., et al. 1990. "Micromechanical modeling of cemented sands under low amplitude oscillations", Geotechnique, Vol. 40, No. 2, pp. 251-263.

TABLE 1
Applicability of Prediction Methods[a]

State	Texture	Saturation	Method
Unfrozen	Coarse Grained	0.015 - 0.100 0.100 - 0.200 0.200 - 1.000 0.000 - 1.000 saturated	Van Rooyen and Winterkorn (except for low-quartz crushed rock) De Vries Johansen Gemant (sandy silt-clay) Johansen, De Vries, Gemant
	Fine Grained	0.000 - 0.100 0.100 - 0.200 0.200 - 1.000 saturated	Johansen (underpredicts by 15%) Johansen (underpredicts by 5%) Johansen Johansen, De Vries, Gemant
Frozen	Coarse Grained	0.100 - 1.000 saturated	Johansen Johansen, De Vries
	Fine Grained	0.000 - 0.900 0.100 - 1.000 saturated	Kersten Johansen Johansen, De Vries

[a]Data from Farouki [13].

TABLE 2
Correlation Coefficients

Soil Type	Frozen / Unfrozen	?$_1$			?$_2$			?$_3$			?$_4$		
		Low	Mean	High	Low	Mean	High	Low	Mean	High	Low	Mean	High
Clay	Frozen	23.5	14.5	14.0	1.73	1.73	1.73	-2.0	-2.5	-3.0	-1.75	-2.0	-2.0
	Unfrozen	33.5	27.0	14.0	2.01	1.84	2.22	-1.6	-1.5	-3.0	-1.31	-0.97	-1.72
Gravel	Frozen	25.4	11.0	11.3	2.01	2.43	2.08	-2.1	-3.0	-2.8	-1.23	-1.6	-0.85
	Unfrozen	16.5	6.5	8.3	2.22	2.63	1.39	-1.9	-3.0	-1.8	-1.1	-1.48	-0.8
Peat	Frozen		12.0			2.77			-2.6			-2.52	
	Unfrozen		28.0			6.00			-1.9			-1.4675	
Sand	Frozen	26.0	10.0	15.0	1.84	1.66	1.18	-1.0	-2.2	-1.8	-0.735	-1.625	-0.44
	Unfrozen	6.4	6.8	6.8	5.55	2.77	3.47	-3.2	-2.9	-7.5	-2.0	-1.5	-2.0
Silt	Frozen	38.0	19.5	18.5	1.66	1.87	1.39	-1.2	-1.8	-2.0	-0.96	-1.53	-1.8
	Unfrozen	28.0	17.0	22.0	2.77	2.77	1.73	-1.0	-2.6	-2.2	-0.6	-1.6	-0.95

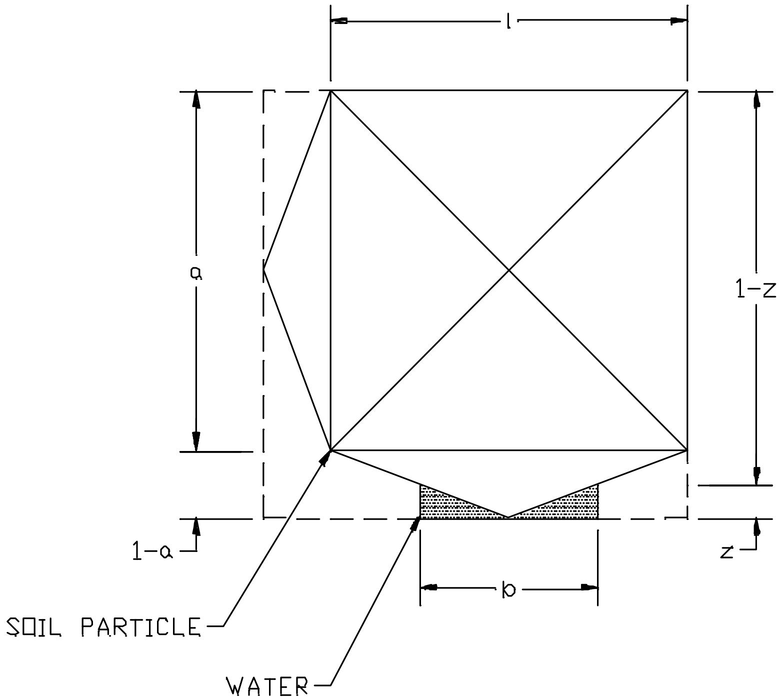


Figure 1: Idealized Soil Particle used in Gemant's Correlation (After Farouki [13]).

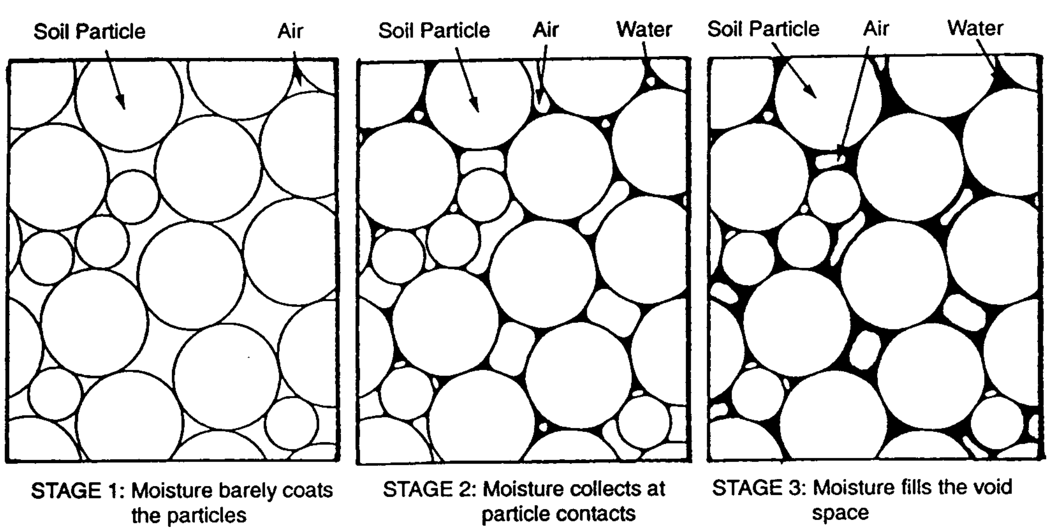
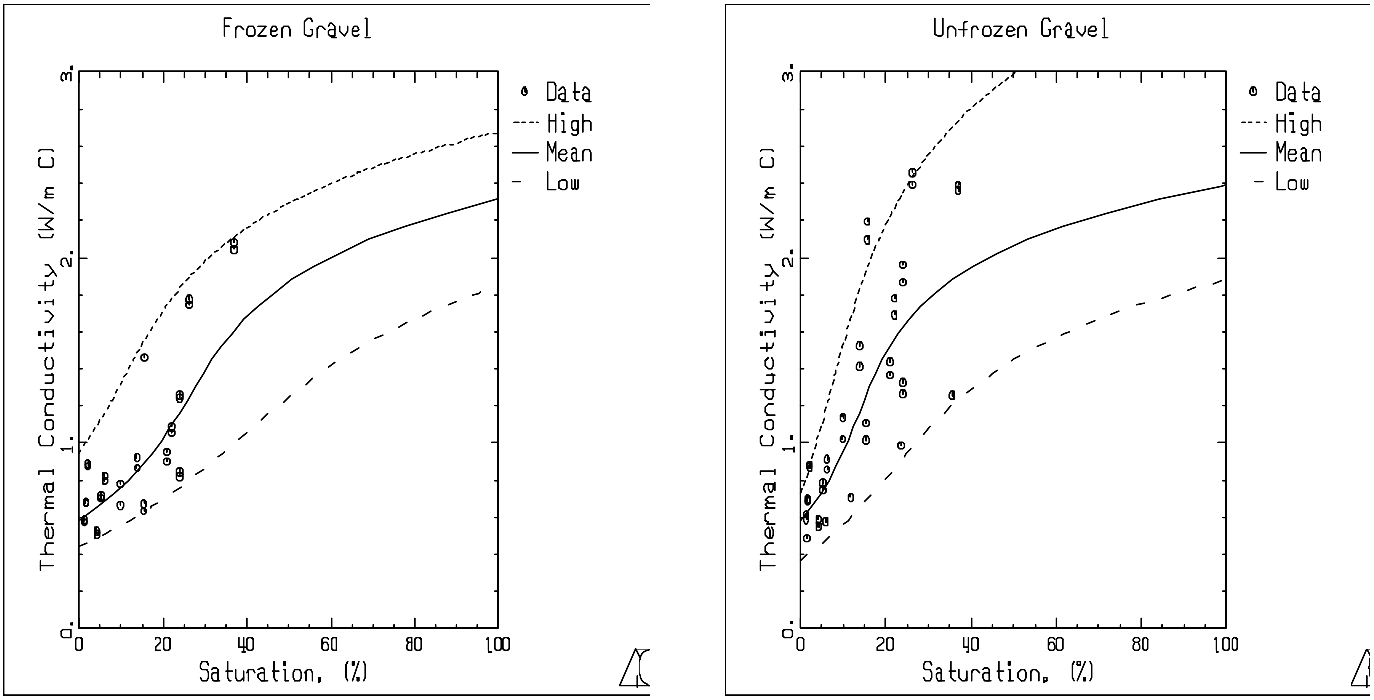


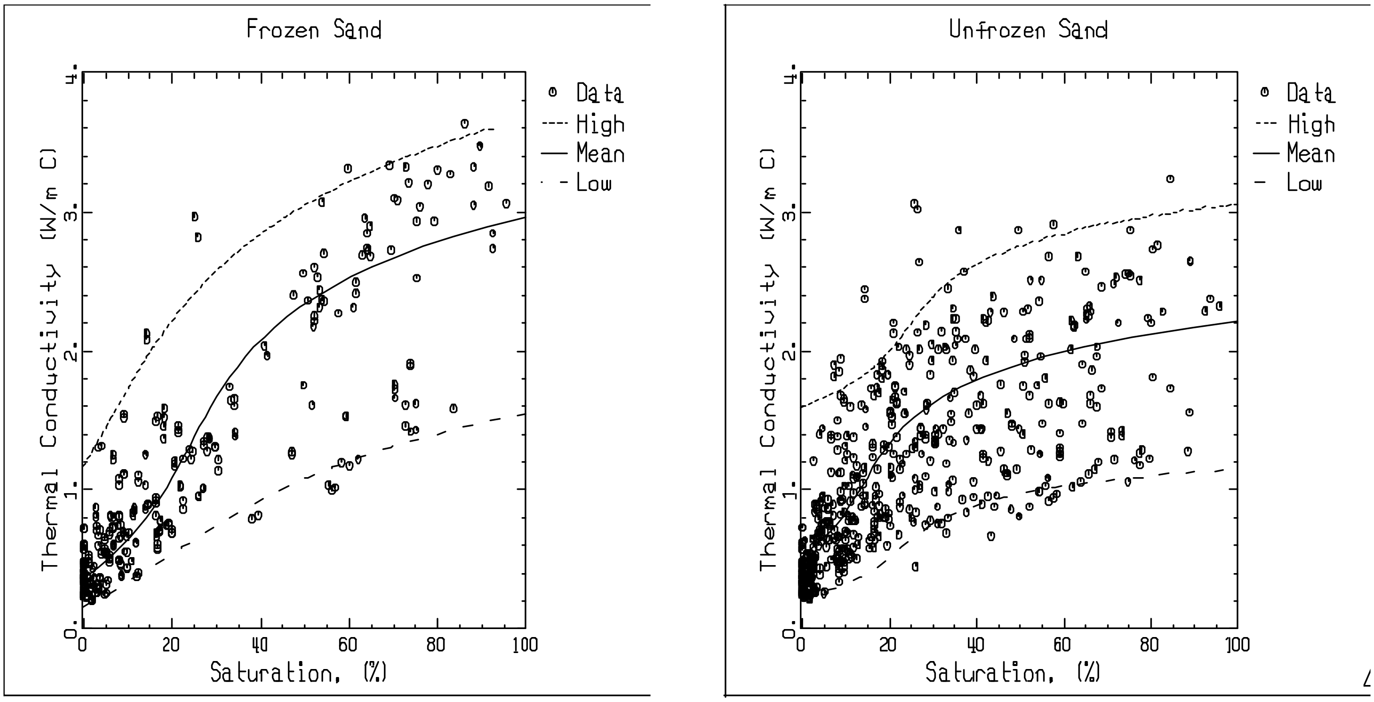
Figure 2: Saturation States of Granular Media:
Stage 1 - Moisture Barely Coats the Particles;
Stage 2 - Moisture Collects at Particle Contacts;
Stage 3 - Moisture Fills the Void Space



(a)

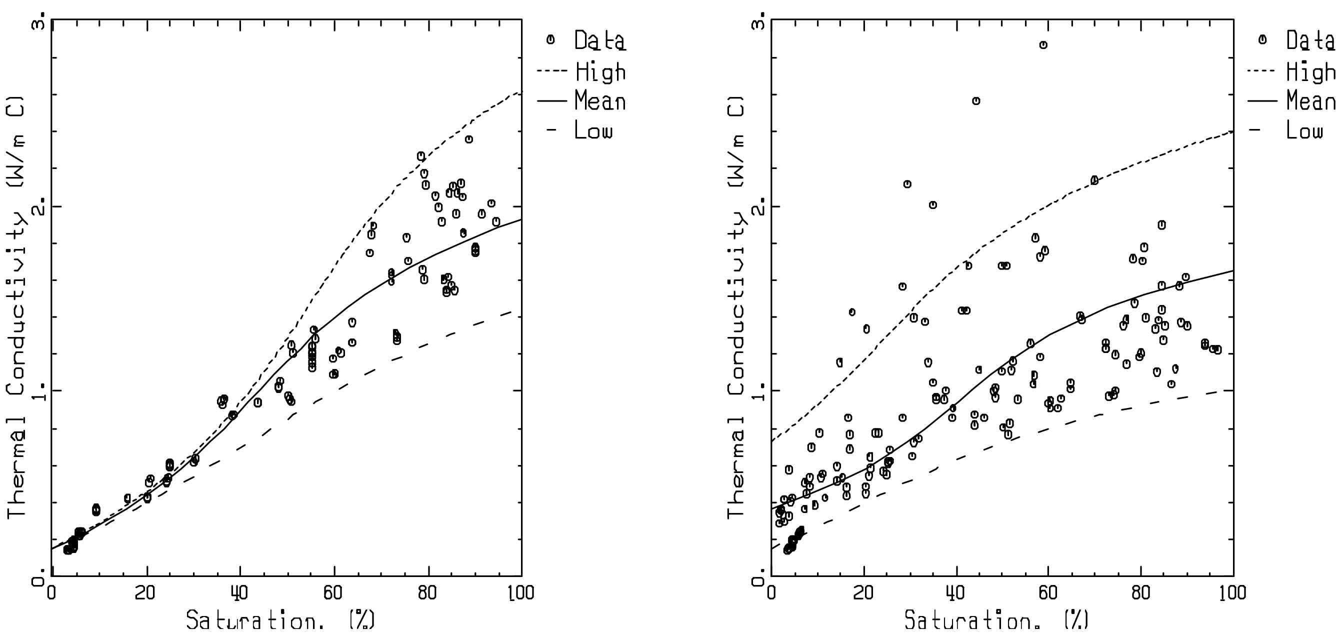
(b)

Figure 3: Thermal Conductivity vs. Saturation for Gravel: (a) Frozen; (b) Unfrozen



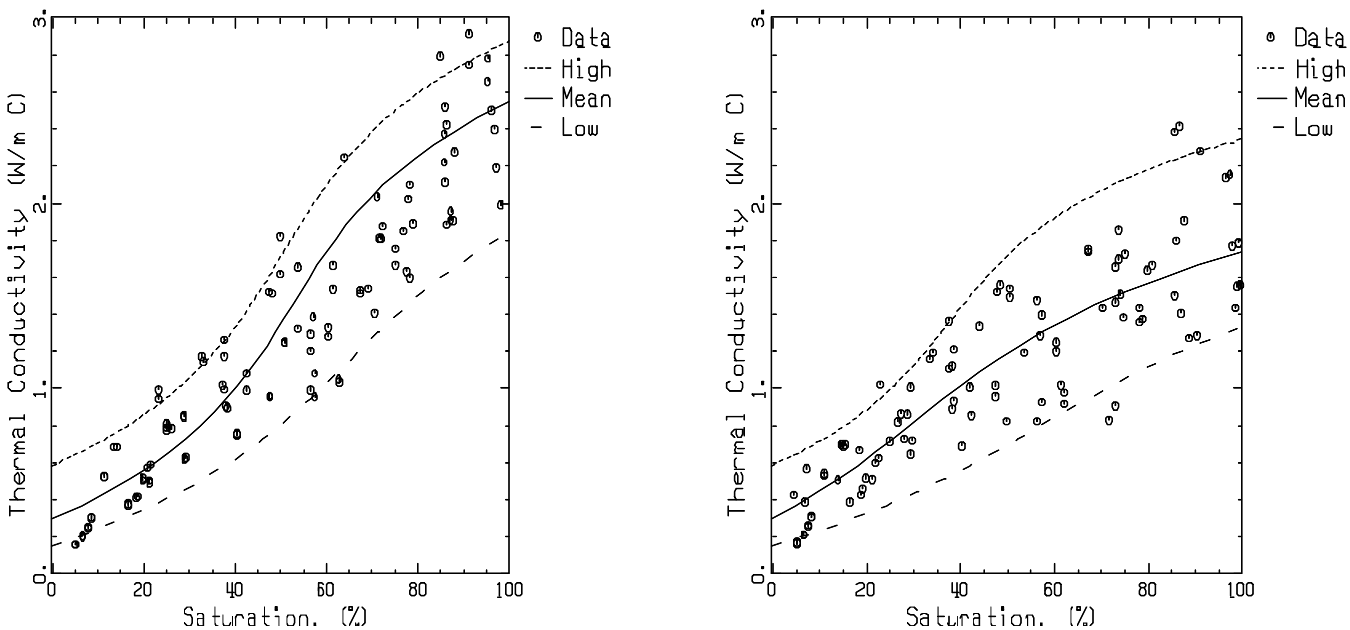
(a) (b)

Figure 4: Thermal Conductivity vs. Saturation for Sand: (a) Frozen; (b) Unfrozen



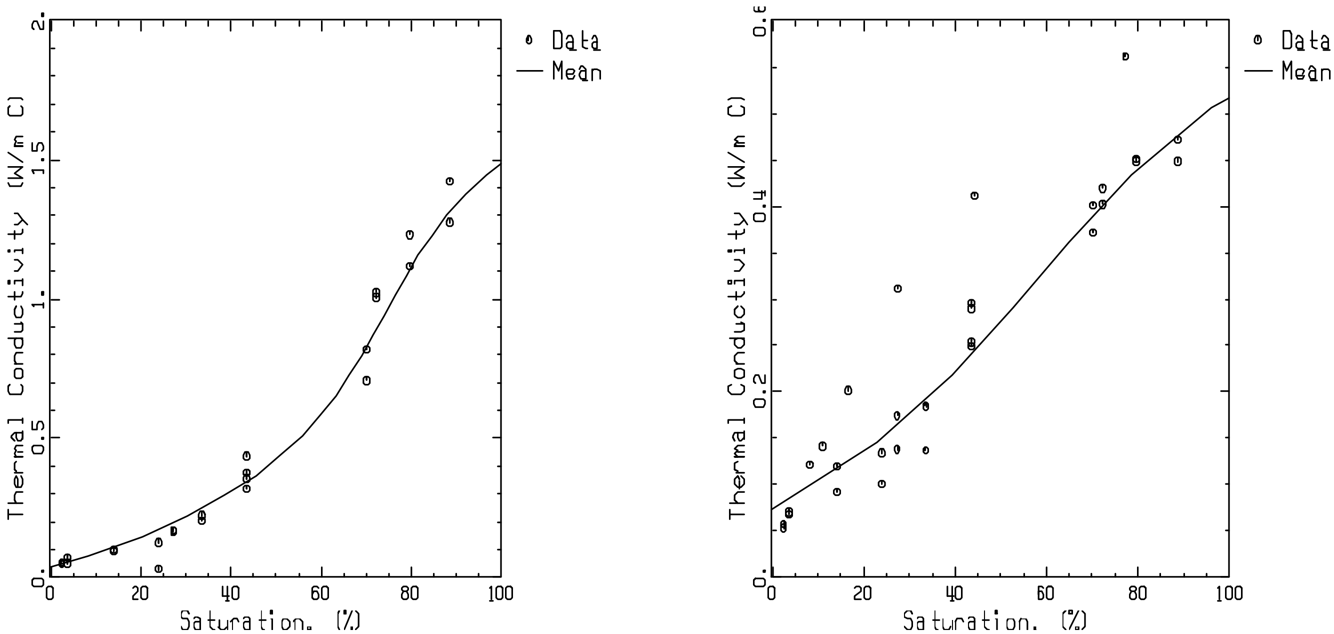
(a) (b)

Figure 5: Thermal Conductivity vs. Saturation for Silt: (a) Frozen; (b) Unfrozen



(a) (b)

Figure 6: Thermal Conductivity vs. Saturation for Clay: (a) Frozen; (b) Unfrozen



(a) (b)

Figure 7: Thermal Conductivity vs. Saturation for Peat: (a) Frozen; (b) Unfrozen

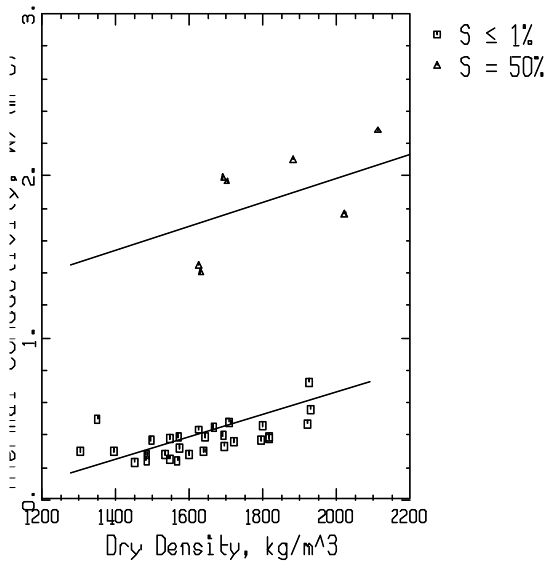


Figure 8: Thermal Conductivity vs. Dry Density for Unfrozen Sand at Two Saturation Levels: S ≤ 1% (☐) and S = 50% (_)

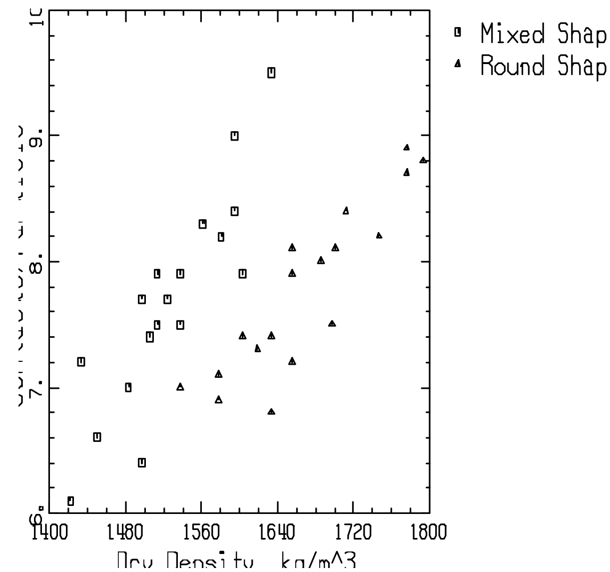


Figure 9: Number of Contacts/Particle vs. Dry Density for Mixed Shaped Particles (☐) and Round Shaped Particles (_)