DEVELOPMENT OF CORRELATIONS FOR SOIL THERMAL CONDUCTIVITY

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ABSTRACT -

Soil thermal conductivity is significantly influenced by saturation and dry density. In this paper, a family of empirical correlations are presented which relate soil thermal conductivity to saturation for five soil types, namely, gravel, sand, silt, clay and peat, in both the frozen and unfrozen states. These correlations were developed from a soil thermal conductivity database which was constructed from measured data available in the literature. The effects of dry density are also examined.

I. ~~Introduction~~

Accurate estimates of a soil's thermal conductivity are of prime importance in the numerical simulation of heat transmission through soils. Soil thermal conductivity estimation methods that could easily be incorporated in computer algorithms will have wide application. They will facilitate the efficient design of ground source heat pump systems and will help ensure the safe design and location of underground storage facilities for nuclear and other hazardous waste materials. Furthermore, the resultant prediction methods will be useful in calculation of heat loss through basements, slabs and crawl spaces. The focus of this paper is to develop correlations of soil thermal conductivity which can be included in numerical heat transfer algorithms.

Factors affecting soil thermal conductivity include moisture content, dry density, mineral composition and temperature [1-5]. Moisture content, by far, has the greatest impact upon soil thermal conductivity. As moisture is added to a soil, a thin water film develops which bridges the gaps between the soil particles. This "bridging" increases the effective contact area between the soil particles, which increases the heat flow and results in higher thermal conductivity. As the voids between the soil particles become completely filled with moisture, the soil thermal conductivity no longer increases with increasing moisture content [3-5].

Soil thermal conductivity also increases with the dry density of the soil. With an increase in soil dry density, more soil particles are packed into a unit volume and the number of contact points between the particles increases. This increase in contact points provides a larger heat flow path and, thus, increases the soil thermal conductivity [3-5].

Soil thermal conductivity also varies with the mineral composition of the soil. For example, sands with a high quartz content generally have a greater thermal conductivity than sands with high contents of plagioclase feldspar and pyroxene [1].

Except at the ice point, soil thermal conductivity varies little with temperature. However, a dramatic change in soil thermal conductivity occurs between the frozen and unfrozen states due to the higher thermal conductivity of ice [1-3].

In this paper correlations are developed for soil thermal conductivity as a function of moisture content. Correlations are presented for various soil types, namely, gravels, sands, silts, clays and peats, in both the frozen and unfrozen states. The effect of dry density on a soil's thermal conductivity is also studied. The developed correlations are based upon measured thermal conductivity data which are available in the literature. The data includes laboratory as well as field measurements.

II. ~~Literature Survey~~

Correlations of soil thermal conductivity have been previously proposed in the literature. Van Rooyen [6], Johansen [7], De Vries [8], Gemant [9], and Kersten [1], among others, have developed correlations for soil thermal conductivity. These correlations vary in complexity, however, each method is limited only to a certain type of soil under specific conditions. Farouki [10] has published a survey of these various correlations which is summarized below.

Van Rooyen's correlation [6], based on data collected from sands and gravels, gives soil thermal conductivity as a function of the degree of saturation, dry density, mineral type, and granulometry. The Van Rooyen method is limited to unfrozen sands and gravels with saturation levels between 1.5% to 10%.

Johansen's correlation [7], which gives soil thermal conductivity as a function of soil saturation, is suitable for both coarse and fine grained soils in the frozen and unfrozen states. However, it is limited to saturations greater than 20%.

The correlation given by De Vries [8] assumes that soil is a two-phase material composed of uniform ellipsoidal particles dispersed in a fluid phase. De Vries' method, which gives soil thermal conductivity as a function of the solid volume fraction and the thermal conductivities of the solid and fluid phases, is applicable to unfrozen coarse soils with saturations between 10% to 20%.

Gemant's correlation [9] is based upon an idealized geometrical model of soil particles with point contacts. It gives soil thermal conductivity as a function of soil dry density, moisture content, apex water (water collected around the contact points), water absorbed as a film around the soil particles, thermal conductivity of the solids, and thermal conductivity of water. Gemant's method gives reasonable results for unfrozen sandy soils only.

Kersten [1] tested many soil types and based his correlations on the empirical data he collected. He produced equations for the thermal conductivity of frozen and unfrozen silt-clay soils and sandy soils as a function of moisture content and dry density. Kersten's correlations give reasonable results only for frozen soils with saturations up to 90%.

Clearly, these existing correlations for soil thermal conductivity are limited in their applicability. Furthermore, they do not offer a unified cogent technique for the estimation of soil thermal conductivity. Therefore, these existing methods cannot be incorporated into numerical heat transfer algorithms.

In contrast, the correlations developed in this paper provide a unified methodology for evaluating soil thermal conductivity. These correlations are applicable to various soil types, namely, gravels, sands, silts, clays and peats, in both the frozen and unfrozen states. Due to their unified format, these new correlations can be readily incorporated into numerical heat transfer algorithms.

III. ~~Development of Correlations~~

In order to develop empirical correlations for soil thermal conductivity, a data base was created from measured data available in the literature. Based upon particle size, soils were classified into five general types, namely, gravel, sand, silt, clay and peat. Thermal conductivities at various dry densities, moisture contents and temperatures were collected for each soil type. To obtain reasonable results, many sources of data were consulted: Kersten [1], Penner et al [2], Salomone and Marlowe [3], De Vries [8], Farouki [10], Andersland and Anderson [11], Nakshabandi and Kohnke [12], and Sawada [13].

From the measured data it is seen that soil thermal conductivity is significantly influenced by saturation and dry density [1-3,8,10-13]. Saturation describes the amount of moisture contained in a soil while dry density refers to the mass of soil particles per unit volume. An increase in either the saturation or dry density of a soil will result in an increase in its thermal conductivity. This paper focusses upon the development of correlations of soil thermal conductivity as a function of saturation. The effects of dry density are also investigated.

A. ~~Basic Definitions~~

An expression for saturation can be derived from the basic definitions of dry density, solid density and moisture content. Dry density, ?$_d$; and solid density, ?$_s$; moisture content, w; and saturation, s, are defined as follows:

$$r_d = \frac{M_S}{V_T} \; ; \quad r_S = \frac{M_S}{V_S} \; ; \quad w = \frac{M_W}{M_S} \; ; \quad S = \frac{V_W}{V_V} \qquad \text{(1a,b,c,d)}$$

In Eq. (1), M_s is the mass of solid soil particles; M_W is the mass of water; V_T is the total volume; V_S is the volume of the solid particles; V_W is the volume of water; and V_V is the volume of void spaces.

Combining Eq. (1a), (1b), (1c), and (1d) yields the following expression for saturation:

$$S = \frac{r_d w}{r_w \left(1 - \frac{r_d}{r_S}\right)} \_ \; 100\% \qquad (2)$$

B. ~~Thermal Conductivity versus Saturation~~

The thermal conductivity of a soil increases in three stages as the saturation level increases. At low saturations, moisture first coats the soil particles. The gaps between the soil particles are not filled rapidly and thus there is a slow increase in thermal conductivity. When the particles are fully coated with moisture, a further increase in the moisture content fills the voids between particles. This increases the heat flow between particles, resulting in a rapid increase in thermal conductivity. Finally, when all the voids are filled, further increasing the moisture content no longer increases the heat flow and the thermal conductivity does not appreciably increase. The model used to describe this behavior is as follows:

$$S = l_1[\sinh(l_2 k + l_3) - \sinh(l_4)] \qquad (3)$$

In Eq. (3), S is the saturation; k is the soil thermal conductivity (Btu·in / ft^2·hr·°F); and $?_1$, $?_2$, $?_3$ and $?_4$ are coefficients which depend upon soil type.

The values of $?_1$ through $?_4$ for each of the five soil types in both the frozen and unfrozen states are given in Table 1. At a saturation of zero, Eq. (3) reduces to: $l_2 k_0 + l_3 = l_4$4. This shows that the coefficient $?_4$ is related to the thermal conductivity of dry soil, k_0.

Figure 1 through Figure 5 present the measured soil thermal conductivity versus saturation data for the five soil types in both the frozen and unfrozen states. The empirical correlations, based upon Eq. (3), are also plotted on Figures 1 through 5. Note that three curves have been given for each soil type (except peat). The upper curve represents the upper limit of the measured data. The middle curve is the mean of the measured data and the lower curve represents the lower limit of the measured data. Due to the small amount of measured data for peaty soils, only a mean correlation is presented. Measured data collected for gravel includes saturations up to approximately 40% and thus, the correlations for gravel are valid only to 40% saturation.

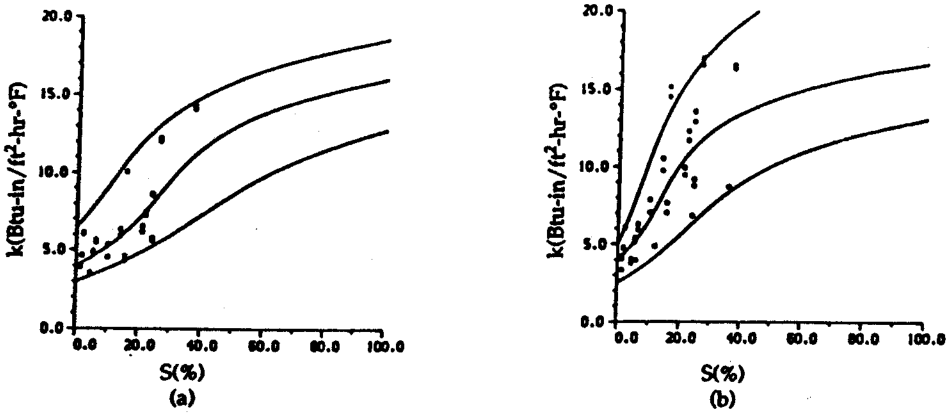
C. ~~Error Analysis~~

The goodness of fit of the mean correlations presented in Figures 1 through 5 was investigated. The difference, Z, between the mean correlation and the measured data was calculated at each data point. A normalized difference, Z^*, was calculated as: $Z^* = (Z - \bar{Z}) / s_Z$5, in which is the mean of the calculated differences and s_Z is the standard deviation of those differences. The cumulative frequency of the normalized difference, Z^*, is compared to a cumulative normal distribution function in Figure 6a through Figure 6h. These plots indicate that the correlations given by Eq. (3) produce a good fit to the measured data. An error distribution for frozen and unfrozen gravel was not investigated due to the limited amount of measured data. The error distributions for peat have limited reliability as there was little measured data available.

TABLE 1

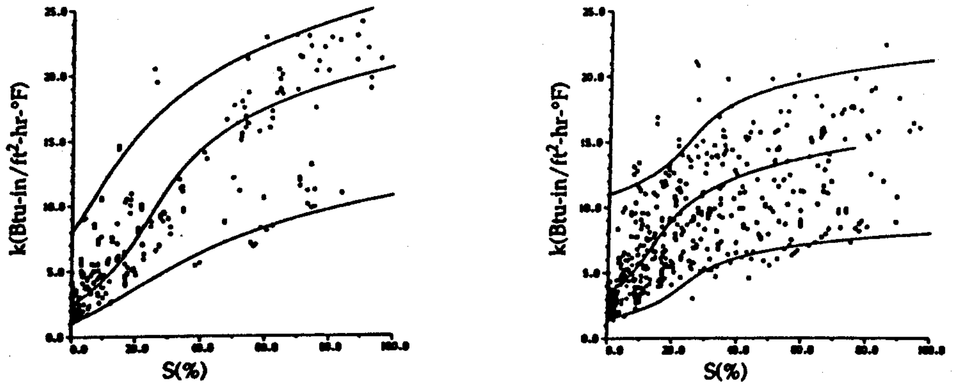
Correlation Coefficients

Soil Type	Frozen Unfrozen	$?_1$			$?_2$			$?_3$			$?_4$		
		Low	Mid	High	Low	Mid	High	Low	Mid	High	Low	Mid	High
Clay	Frozen	23.5	14.5	14.0	0.25	0.25	0.25	-2.0	-2.5	-3.0	-1.75	-2.0	-2.0
	Unfrozen	33.5	27.0	14.0	0.29	0.265	0.32	-1.6	-1.5	-3.0	-1.31	-0.97	-1.72
Gravel	Frozen	25.4	11.0	11.3	0.29	0.35	0.3	-2.1	-3.0	-2.8	-1.23	-1.6	-0.85
	Unfrozen	16.5	6.5	8.3	0.32	0.38	0.2	-1.9	-3.0	-1.8	-1.1	-1.48	-0.8
Peat	Frozen		12.0			0.4			-2.6			-2.52	
	Unfrozen		28.0			0.865			-1.9			-1.4675	
Sand	Frozen	26.0	10.0	15.0	0.265	0.24	0.17	-1.0	-2.2	-1.8	-0.735	-1.625	-0.44
	Unfrozen	6.4	6.8	6.8	0.8	0.4	0.5	-3.2	-2.9	-7.5	-2.0	-1.5	-2.0
Silt	Frozen	38.0	19.5	18.5	0.24	0.27	0.2	-1.2	-1.8	-2.0	-0.96	-1.53	-1.8
	Unfrozen	28.0	17.0	22.0	0.4	0.4	0.25	-1.0	-2.6	-2.2	-0.6	-1.6	-0.95



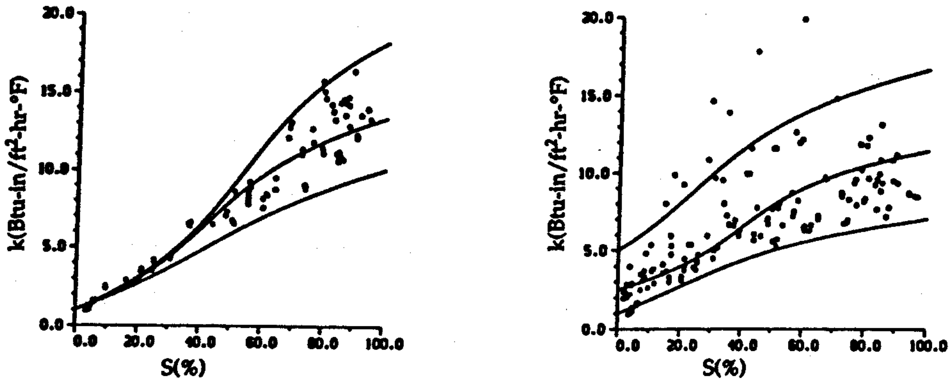
(a) (b)

FIG. 1
Thermal conductivity vs. saturation for gravel: (a) frozen; (b) unfrozen.



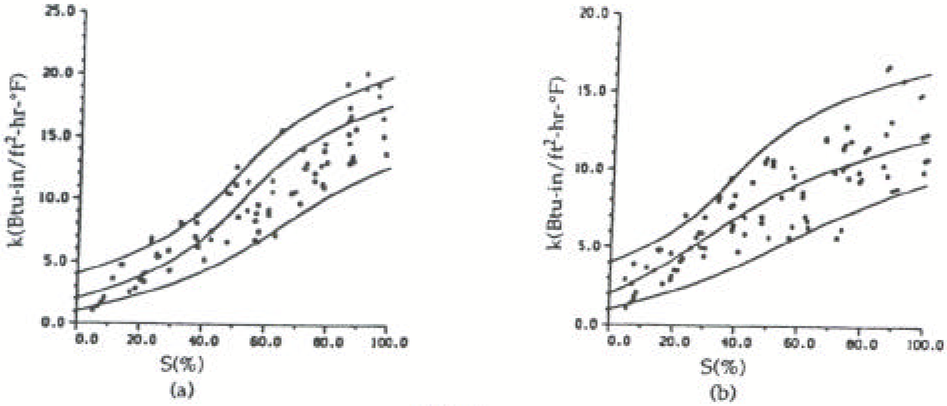
(a) (b)

FIG. 2
Thermal conductivity vs. saturation for sand: (a) frozen; (b) unfrozen.



(a) (b)

FIG. 3
Thermal conductivity vs. saturation for silt: (a) frozen; (b) unfrozen.

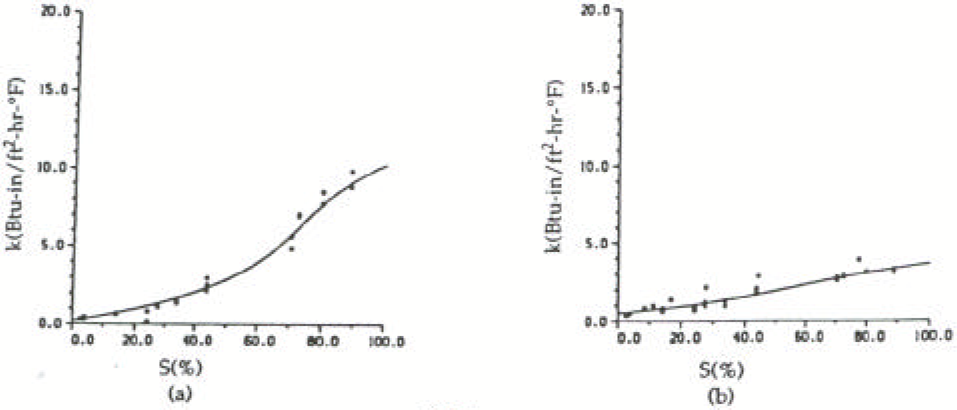


(a)

(b)

FIG. 4

Thermal conductivity vs. saturation for clay: (a) frozen; (b) unfrozen.



(a)

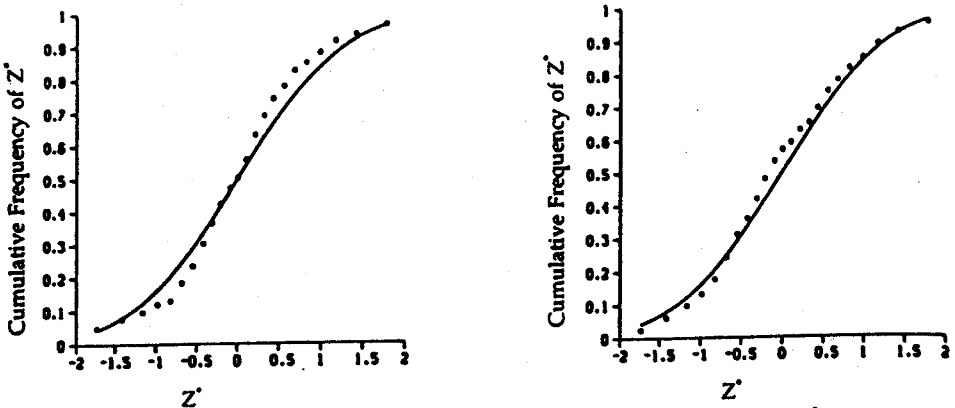
(b)

FIG. 5

Thermal conductivity vs. saturation for peat: (a) frozen; (b) unfrozen.

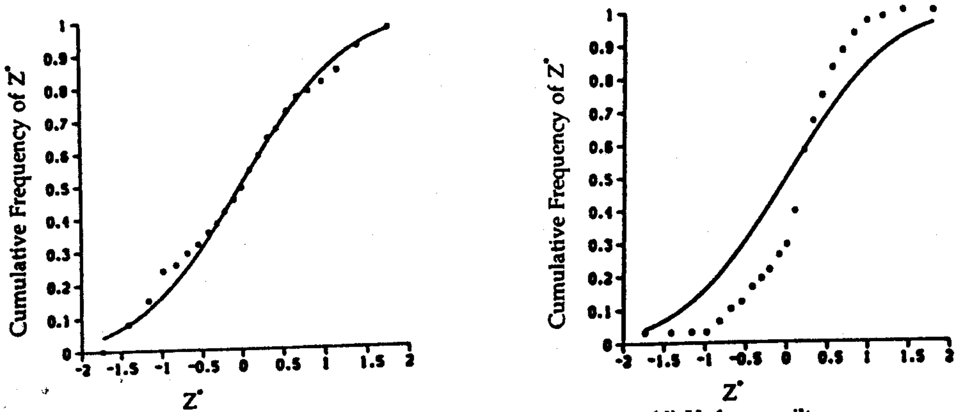
D. Effects of Dry Density

At any given saturation level, the soil thermal conductivity exhibits considerable variation as shown in Figure 1 through Figure 5. This variation is due, in part, to differences in dry density. Soil thermal conductivity increases with the dry density of the soil. As shown in Figure 7, the relationship between dry density and thermal conductivity is linear at a given level of saturation. As shown in Figure 8, when the dry density increases, the number of contact points between the soil particles increases [14]. This increase in contact points results in an increase in the effective heat flow area which causes an increase in soil thermal conductivity.



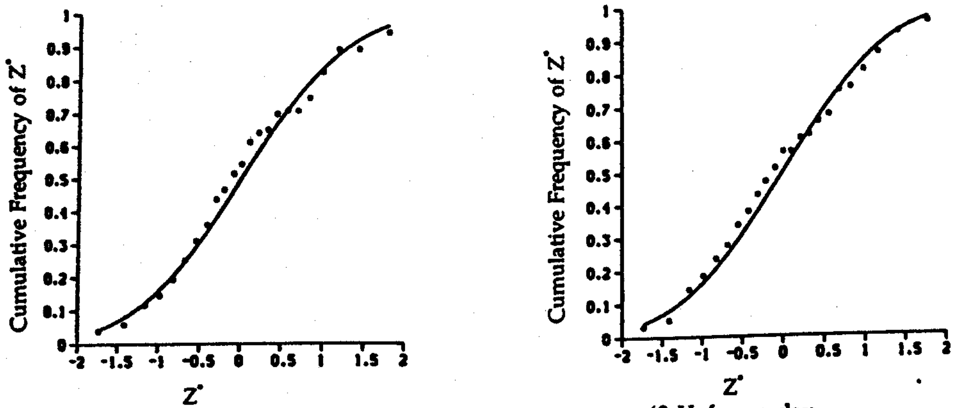
(a) Frozen sand.

(b) Unfrozen sand.



(c) Frozen silt.

(d) Unfrozen silt.

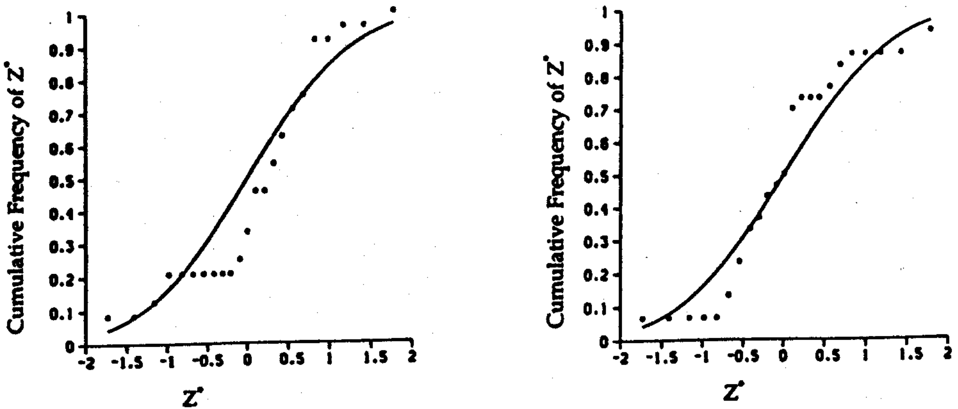


(e) Frozen clay.

(f) Unfrozen clay.

FIG. 6

Cumulative frequency of Z^* vs. Z^*. The solid line is the cumulative normal distribution which is shown for comparison.



(g) Frozen peat.

(h) Unfrozen peat.

FIG. 6 (continued)

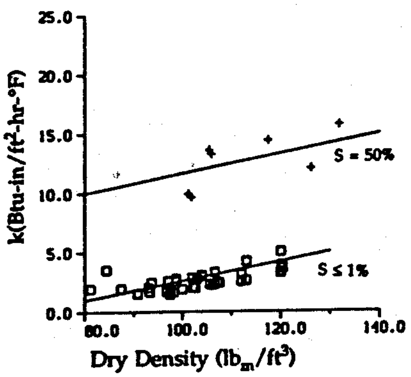


FIG. 7
Thermal conductivity vs. dry density for unfrozen sand at two saturation levels: s ≤ 1%(□) and s = 50% (+).

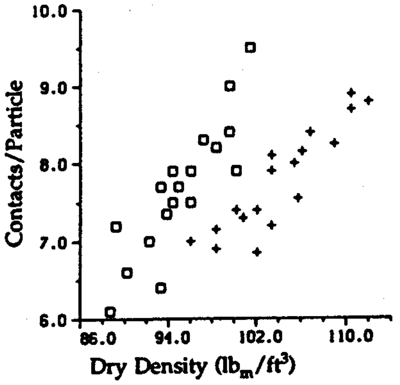


FIG. 8
Number of contacts/particle vs. dry density for mixed shaped particles (□) and round shaped particles (+).

At a given dry density and saturation level, the scatter of soil thermal conductivity is due, in part, to the shape of the soil particles. Figure 8 shows that particle shape influences how well particles can be packed together. Soils with relatively flat surfaces have a larger number of contact points, thus resulting in greater thermal conductivity as compared to soils composed of round particles which have a smaller number of contact points.

IV. ~~Conclusions~~

In conclusion, a soil thermal conductivity database has been constructed from measured data available in the literature. From this database, a family of empirical correlations have been developed which relate soil thermal conductivity to saturation for five soil types, namely, gravel, sand, silt, clay, and peat, in both the frozen and unfrozen states. The error analysis shows that these correlations provide a good fit to the measured data available in the literature. The unified format of these correlations make them readily adaptable to numerical heat transfer algorithms. In addition, dry density has been shown to significantly influence the scatter of soil thermal conductivity at a given saturation level.

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