# Evaluation of Semi-Analytical/Empirical Freezing Time Estimation Methods Part I: Regularly Shaped Food Items

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*The freezing of food is one of the most significant applications of refrigeration. Numerous semi-analytical/ empirical methods for predicting food freezing times have been proposed. Therefore, a quantitative evaluation of selected semi-analytical/empirical food freezing time prediction methods is provided in two parts. This report focuses on methods that apply to regularly shaped food items, while Part II covers techniques that apply to irregularly shaped food items. The performance of these various methods is quantitatively evaluated by comparing their numerical results to a comprehensive experimental freezing time data set compiled from the literature.*

## INTRODUCTION

Preservation of food is one of the most significant applications of refrigeration. It is known that the freezing of food effectively reduces the activity of microorganisms and enzymes, thus retarding deterioration. In addition, crystallization of water reduces the amount of liquid water in food items and inhibits microbial growth (Heldman 1975).

In order for food freezing operations to be cost effective, it is necessary to optimally design the refrigeration equipment to fit the specific requirements of the particular freezing application. The design of such refrigeration equipment requires estimation of the freezing times of foods, as well as the corresponding refrigeration loads.

Numerous methods for predicting food freezing times have been proposed. The designer is thus faced with the challenge of selecting an appropriate estimation method from the plethora of available methods. This paper focuses on those methods that apply to regularly shaped food items, while Part II covers techniques that apply to irregularly shaped food items. The performance of these various methods is quantitatively evaluated by comparing their numerical results to a comprehensive experimental freezing time data set compiled from the literature. This evaluation provides a consistent basis for the comparison of semi-analytical/empirical freezing time estimation methods. It also establishes confidence in the use of these methods and facilitates the designer's selection of an appropriate freezing time estimation method.

## THERMODYNAMICS OF THE FREEZING PROCESS

The freezing of food is a complex process. Prior to freezing, sensible heat must be removed from the food to decrease its temperature from the initial temperature to the initial freezing point of the food. This initial freezing point is somewhat lower than the freezing point of pure water due to dissolved substances in the moisture within the food. At the initial freezing point, a portion of the water within the food crystallizes and the remaining solution becomes more concen-

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trated. Thus, the freezing point of the unfrozen portion of the food is further reduced. As the temperature continues to decrease, the formation of ice crystals increases the concentration of the solutes in solution and depresses the freezing point further. Thus, it is evident that during the freezing process, the ice and water fractions in the frozen food depend on temperature. Since the thermophysical properties of ice and liquid water are quite different, the corresponding properties of the frozen food are temperature dependent. Therefore, due to these complexities, it is not possible to derive exact analytical solutions for the freezing times of foods.

Theoretically, the freezing of food can be described via the Fourier heat conduction equation:

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c}\left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)\right] \quad (1)$$

where $T$ is temperature, $t$ is time, $\rho$ is the density of the food, $c$ is the specific heat of the food, $k$ is the thermal conductivity of the food and $x$, $y$, and $z$ are the coordinate directions. For ideal, regularly shaped food items with constant thermophysical properties, uniform initial conditions, constant external conditions and prescribed surface temperature or convection boundary conditions, exact analytical solutions for Equation (1) exist, which permit freezing time estimation. However, for practical freezing processes, food items are generally irregularly shaped with temperature dependent thermophysical properties, and therefore, it is not possible to derive exact analytical solutions for their freezing times.

Numerical estimates of food freezing times can be obtained using appropriate finite element or finite difference computer programs. However, the effort required to perform this task makes it impractical for the design engineer. In addition, two-dimensional and three-dimensional simulations require time consuming data preparation and significant computing time. Hence, the majority of the research effort to date has been in the development of semi-analytical/empirical food freezing time prediction methods which make use of simplifying assumptions.

These semi-analytical/empirical freezing time prediction methods fall into two main categories. Methods in the first category apply to food items that have the following regular shapes:

- Infinite slabs
- Infinite circular cylinders
- Spheres

Methods in the second category, discussed in Part II, apply to food items that have irregular shapes. These methods require a two-step procedure in which the freezing time is first estimated by using one of the methods applicable to regularly shaped food items. Thus, freezing time estimation for both regularly and irregularly shaped food items requires the use of the methods described here.

## FREEZING TIME ESTIMATION METHODS

### Plank's Equation

The most widely known method for estimating the freezing times of foods is that developed by Plank (1913, 1941). In this method, the following is assumed:

- Only convective heat transfer occurs between the food item and the surrounding cooling medium
- The temperature of the food item is its initial freezing temperature and that this temperature is constant throughout the freezing process
- Thermal conductivity for the frozen region is constant

Plank's freezing time estimation method is given as follows:

$$t = \frac{L_f}{T_f - T_m}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right) \tag{2}$$

where $L_f$ is the volumetric latent heat of fusion, $T_f$ is the initial freezing temperature of the food, $T_m$ is the freezing medium temperature, $D$ is the thickness of the slab or the diameter of the sphere or infinite cylinder, $h$ is the surface heat transfer coefficient, $k_s$ is the thermal conductivity of the fully frozen food, and $P$ and $R$ are geometric factors. For the infinite slab, $P = 1/2$ and $R = 1/8$. For a sphere, $P$ and $R$ are 1/6 and 1/24, respectively; and for an infinite cylinder, $P = 1/4$ and $R = 1/16$.

The geometric factors, $P$ and $R$ provide insight as to the effect of shape on freezing time. Plank's shape factors indicate that an infinite slab of thickness $D$, an infinite cylinder of diameter $D$, and a sphere of diameter $D$, if exposed to the same conditions, would have freezing times in the ratio of 6:3:2. Hence, a cylinder will freeze in half the time of a slab and a sphere will freeze in one-third the time of a slab.

Various researchers have noted that Plank's method does not accurately predict the freezing times of foods. This is due, in part, to the fact that Plank's method assumes that food freezing takes place at a constant temperature, and not over a range of temperatures as is the case in actual food freezing processes. In addition, the thermal conductivity of the frozen food is assumed constant, but in reality, the thermal conductivity varies greatly during freezing. Another limitation of Plank's equation is that it neglects the removal of sensible heat above the freezing point. However, Plank's method does have the advantage of being a simple model for predicting food freezing time. Subsequently, researchers have focused on development of improved semi-analytical/empirical cooling and freezing time estimation methods that account for precooling and subcooling times, non-constant thermal properties, and phase change over a range of temperatures.

## Modifications to Plank's Equation

Cleland and Earle (1977, 1979) improved on Plank's model by incorporating corrections to account for the removal of sensible heat both above and below the initial freezing point of the food as well as temperature variation during freezing. Regression equations were developed to estimate the $P$ and $R$ for infinite slabs, infinite cylinders, and spheres. In these regression equations, the effects of surface heat transfer, precooling and final subcooling are accounted for by means of the Biot number, the Plank number, and the Stefan number, respectively.

The Biot number is defined as follows:

$$\mathrm{Bi} = hD/k \tag{3}$$

where $h$ is the surface heat transfer coefficient, $D$ is the characteristic dimension and $k$ is the thermal conductivity. In the literature on food freezing, it is accepted that the characteristic dimension $D$ is defined to be twice the shortest distance from the thermal center of a food item to its surface. For an infinite slab, $D$ is the thickness. For an infinite cylinder or a sphere, $D$ is the diameter. These definitions will be adopted for this paper, unless otherwise noted.

In general, the Plank number is defined as follows:

$$\mathrm{Pk} = C_l(T_i - T_f)/\Delta H \tag{4}$$

where $C_l$ is the volumetric specific heat of the unfrozen phase and $\Delta H$ is the volumetric enthalpy change of the food between $T_f$ and the final food temperature. The Stefan number is similarly defined as follows:

$$\text{Ste} = C_s(T_f - T_m)/\Delta H \tag{5}$$

where $C_s$ is the volumetric specific heat of the frozen phase.

In the method of Cleland and Earle (1977, 1979), food freezing times are calculated with a modified version of Plank's equation. Plank's original geometric factors $P$ and $R$ are replaced with the modified values given in Table 1, and the latent heat $L_f$ in Plank's equation is replaced with the volumetric enthalpy change of the food $\Delta H_{10}$, between the freezing temperature $T_f$ and the final center temperature, assumed to be −10°C. As shown in Table 1, $P$ and $R$ are functions of the Plank number and the Stefan number. Both of these parameters should be evaluated using the enthalpy change $\Delta H_{10}$. Thus, the modified Plank equation takes the following form:

$$t = \frac{\Delta H_{10}}{T_f - T_m}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right) \tag{6}$$

where $k_s$ is the thermal conductivity of the fully frozen food.

**Table 1. Expressions for *P* and *R* from Cleland and Earle (1977, 1979)**

| **Shape** | ***P* and *R* Expressions** |
|---|---|
| Infinite slab | $P = 0.5072 + 0.2018\text{Pk} + \text{Ste}\left(0.3224\text{Pk} + \frac{0.0105}{\text{Bi}} + 0.0681\right)$ |
| | $R = 0.1684 + \text{Ste}(0.2740\text{Pk} - 0.0135)$ |
| Infinite cylinder | $P = 0.3751 + 0.0999\text{Pk} + \text{Ste}\left(0.4008\text{Pk} + \frac{0.0710}{\text{Bi}} - 0.5865\right)$ |
| | $R = 0.0133 + \text{Ste}(0.0415\text{Pk} - 0.3957)$ |
| Sphere | $P = 0.1084 + 0.0924\text{Pk} + \text{Ste}\left(0.231\text{Pk} - \frac{0.3114}{\text{Bi}} + 0.6739\right)$ |
| | $R = 0.0784 + \text{Ste}(0.0386\text{Pk} - 0.1694)$ |

Cleland and Earle (1984) noted that these prediction formulae do not perform as well when applied to situations with final center temperatures other than −10°C. They proposed the following modified form of Equation (6) to account for different final center temperatures.

$$t = \frac{\Delta H_{10}}{T_f - T_m}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right)\left[1 - \frac{1.65\text{Ste}}{k_s}\ln\left(\frac{T_c - T_m}{T_{ref} - T_m}\right)\right] \tag{7}$$

where $T_{ref}$ is −10°C, $T_c$ is the final product center temperature, and $\Delta H_{10}$ is the volumetric enthalpy difference between the initial freezing temperature $T_f$, and −10°C. The values of $P$ and $R$, the Plank number, and the Stefan number should be evaluated using $\Delta H_{10}$, as previously discussed.

Hung and Thompson (1983) also improved on Plank's equation to develop an alternative freezing time estimation method for infinite slabs. Their equation incorporates the volumetric change in enthalpy $\Delta H_{18}$ for the freezing process as well as a weighted average temperature difference between the initial temperature of the food and the freezing medium temperature. This weighted average temperature difference $\Delta T$ is given as follows:

$$\Delta T = (T_f - T_m) + \frac{(T_i - T_f)^2 \frac{C_1}{2} - (T_f - T_c)^2 \frac{C_s}{2}}{\Delta H_{18}} \tag{8}$$

where $T_c$ is the final center temperature of the food and $\Delta H_{18}$ is the enthalpy change of the food between the initial temperature and the final center temperature, assumed to be −18°C. Empirical equations were developed to estimate the geometric factors $P$ and $R$ for infinite slabs as follows:

$$P = 0.7306 - 1.083\text{Pk} + \text{Ste}\left(15.40U - 15.43 + 0.01329\frac{\text{Ste}}{\text{Bi}}\right) \tag{9}$$

$$R = 0.2079 - 0.2656U\,\text{Ste} \tag{10}$$

where $U = \Delta T/(T_f - T_m)$. In these expressions, the Plank number and the Stefan number should be evaluated using the enthalpy change $\Delta H_{18}$. The freezing time prediction model is written as:

$$t = \frac{\Delta H_{18}}{\Delta T}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right) \tag{11}$$

Cleland and Earle (1984) found that by applying their correction factor to the Hung and Thompson model, the prediction accuracy of the model was improved for final temperatures other than −18°C. The correction to the Hung and Thompson model is as follows:

$$t = \frac{\Delta H_{18}}{\Delta T}\left(\frac{PD}{h} + \frac{RD^2}{k_s}\right)\left[1 - \frac{1.65\text{Ste}}{k_s}\ln\left(\frac{T_c - T_m}{T_{ref} - T_m}\right)\right] \tag{12}$$

where $T_{ref}$ is −18°C, $T_c$ is the product final center temperature, and $\Delta H_{18}$ is the volumetric enthalpy change between the initial temperature $T_i$ and −18°C. The weighted average temperature difference $\Delta T$, the Plank number, and the Stefan number should be evaluated using $\Delta H_{18}$.

## Precooling, Phase Change, and Subcooling Time Calculations

Numerous researchers have taken a different approach to account for the effects of sensible heat removal above and below the initial freezing point. In these methods, the total freezing time, $t$, is the sum of the precooling, phase change and subcooling times:

$$t = t_1 + t_2 + t_3 \tag{13}$$

where $t_1$, $t_2$, and $t_3$ are the precooling, phase change, and subcooling times, respectively.

Lacroix and Castaigne (1987a, 1987b, 1988) suggested the use of $f$ and $j$ factors to determine the precooling and subcooling times of foods. They presented equations, given in Tables 2, 3, and 4, for estimating the values of $f$ and $j$ for infinite slabs, infinite cylinders, and spheres. Note that throughout the method presented by Lacroix and Castaigne, the Biot number ($\text{Bi} = hL/k$), is based on the shortest distance between the thermal center of the food item and its surface $L$, not twice that distance.

**Table 2. Expressions for Estimating $f$ and $j_c$ for Thermal Center Temperature of Infinite Slabs**
(Lacroix and Castaigne 1987a)

| Biot Number Range | Equations for $f$ and $j$ factors |
|---|---|
| $Bi \le 0.1$ | $\frac{f\alpha}{L^2} = \frac{\ln 10}{Bi}$<br>$j_c = 1.0$ |
| $0.1 < Bi \le 100$ | $\frac{f\alpha}{L^2} = \frac{\ln 10}{u^2}$<br>$j_c = \frac{2 \sin u}{u + \sin u \cos u}$<br>where<br>$u = 0.860972 + 0.312133\ln(Bi) + 0.007986[\ln(Bi)]^2 - 0.016192[\ln(Bi)]^3 - 0.001190[\ln(Bi)]^4 + 0.000581[\ln(Bi)]^5$ |
| $Bi > 100$ | $\frac{f\alpha}{L^2} = 0.9332$<br>$j_c = 1.273$ |

**Table 3. Expressions for Estimating $f$ and $j_c$ for Thermal Center Temperature of Infinite Cylinders**
(Lacroix and Castaigne 1987a)

| Biot Number Range | Equations for $f$ and $j$ factors |
|---|---|
| $Bi \le 0.1$ | $\frac{f\alpha}{L^2} = \frac{\ln 10}{2Bi}$<br>$j_c = 1.0$ |
| $0.1 < Bi \le 100$ | $\frac{f\alpha}{L^2} = \frac{\ln 10}{v^2}$<br>$j_c = \frac{2J_1(v)}{v[J_0^2(v) - J_1^2(v)]}$<br>where<br>$v = 1.257493 + 0.487941\ln(Bi) + 0.025322[\ln(Bi)]^2 - 0.026568[\ln(Bi)]^3 - 0.002888[\ln(Bi)]^4 + 0.001078[\ln(Bi)]^5$<br>and $J_0(v)$ and $J_1(v)$ are zero order and first order Bessel functions, respectively. |
| $Bi > 100$ | $\frac{f\alpha}{L^2} = 0.3982$<br>$j_c = 1.6015$ |

**Table 4. Expressions for Estimating $f$ and $j_c$ for Thermal Center Temperature of Spheres**
(Lacroix and Castaigne 1987a)

| Biot Number Range | Equations for $f$ and $j$ factors |
|---|---|
| $\mathrm{Bi} \le 0.1$ | $\frac{f\alpha}{L^2} = \frac{\ln 10}{3\mathrm{Bi}}$ <br> $j_c = 1.0$ |
| $0.1 < \mathrm{Bi} \le 100$ | $\frac{f\alpha}{L^2} = \frac{\ln 10}{w^2}$ <br> $j_c = \frac{2(\sin w - w\cos w)}{w - \sin w \cos w}$ <br> where <br> $w = 1.573729 + 0.642906\ln(\mathrm{Bi}) + 0.047859[\ln(\mathrm{Bi})]^2 - 0.03553[\ln(\mathrm{Bi})]^3 - 0.004907[\ln(\mathrm{Bi})]^4 + 0.001563[\ln(\mathrm{Bi})]^5$ |
| $\mathrm{Bi} > 100$ | $\frac{f\alpha}{L^2} = 0.2333$ <br> $j_c = 2.0$ |

Lacroix and Castaigne (1987a, 1987b, 1988) gave the following expression for estimating the precooling time $t_1$:

$$t_1 = f_1 \log\left[j_1 \frac{T_m - T_i}{T_m - T_f}\right] \tag{14}$$

where $T_m$ is the coolant temperature, $T_i$ is the initial temperature of the food, and $T_f$ is the initial freezing point of the food. The $f_1$ and $j_1$ factors are determined from a Biot number that is calculated using an average thermal conductivity. This average thermal conductivity is based on the thermal conductivity of the unfrozen food, and the thermal conductivity of the frozen food evaluated at the temperature $(T_f + T_m)/2$.

A similar expression is given for estimating the subcooling time $t_3$:

$$t_3 = f_3 \log\left[j_3 \frac{T_m - T_f}{T_m - T_c}\right] \tag{15}$$

where $T_c$ is the final temperature at the center of the food item. The $f_3$ and $j_3$ factors are determined from a Biot number that is calculated using the thermal conductivity of the frozen food evaluated at the temperature $(T_f + T_m)/2$.

Lacroix and Castaigne (1987a, 1987b, 1988) model the phase change time $t_2$ with Plank's equation:

$$t_2 = \frac{L_f D^2}{(T_f - T_m)k_c}\left(\frac{P}{2\mathrm{Bi}_c} + R\right) \tag{16}$$

where $L_f$ is the volumetric latent heat of fusion of the food, $P$ and $R$ are the original Plank geometric shape factors, $k_c$ is the thermal conductivity of the frozen food at the temperature $(T_f + T_m)/2$, and $\mathrm{Bi}_c$ is the Biot number for the subcooling period ($\mathrm{Bi}_c = hL/k_c$).

Lacroix and Castaigne (1987a, 1987b) noted that by making adjustments to Plank's geometric factors $P$ and $R$ better agreement between predicted freezing times and experimental data was obtained. Using regression analysis, Lacroix and Castaigne suggested that the geometric factors should be as follows.

For infinite slabs:

$$P = 0.51233 \tag{17}$$

$$R = 0.15396 \tag{18}$$

For infinite cylinders:

$$P = 0.27553 \tag{19}$$

$$R = 0.07212 \tag{20}$$

For spheres:

$$P = 0.19665 \tag{21}$$

$$R = 0.03939 \tag{22}$$

Pham (1984) also devised a food freezing time estimation method, similar to Plank's equation, in which sensible heat effects are considered by calculating precooling, phase change and subcooling times separately. In addition, Pham suggested the use of a mean freezing point, which is assumed to be 1.5 K below the initial freezing point of the food, to account for freezing which takes place over a range of temperatures. Pham's freezing time estimation method is stated in terms of the volume and surface area of the food item and is therefore applicable to food items of any shape. This method is given as:

$$t_i = \frac{Q_i}{hA_s \Delta T_{mi}}\left(1 + \frac{\mathrm{Bi}_i}{k_i}\right) \quad ; \quad i = 1, 2, 3 \tag{23}$$

where $t_1$ is the precooling time, $t_2$ is the phase change time, and $t_3$ is the subcooling time with the remaining variables defined as shown in Table 5.

Pham (1986) significantly simplified the previous freezing time estimation method (Pham 1984) to yield a single equation which includes precooling, phase change and subcooling:

$$t = \frac{V}{hA_s}\left(\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right)\left(1 + \frac{\mathrm{Bi}_s}{4}\right) \tag{24}$$

in which

$$\Delta H_1 = C_1(T_i - T_{fm}) \tag{25a}$$

$$\Delta H_2 = L_f + C_s(T_{fm} - T_c) \tag{25b}$$

$$\Delta T_1 = \frac{T_i + T_{fm}}{2} - T_m \tag{26a}$$

$$\Delta T_2 = T_{fm} - T_m \tag{26b}$$

**Table 5. Definition of Variables for Freezing Time Estimation Method of Pham (1984)**

| Process | Variables |
|---|---|
| Precooling | $i = 1$<br>$k_1 = 6$<br>$Q_1 = C_1(T_i - T_{fm})V$<br>$\mathrm{Bi}_1 = (\mathrm{Bi}_l + \mathrm{Bi}_s)/2$<br>$\Delta T_{m1} = \dfrac{(T_i - T_m) - (T_{fm} - T_m)}{\ln\left[\dfrac{T_i - T_m}{T_{fm} - T_m}\right]}$ |
| Phase change | $i = 2$<br>$k_2 = 4$<br>$Q_2 = L_f V$<br>$\mathrm{Bi}_2 = \mathrm{Bi}_s$<br>$\Delta T_{m2} = T_{fm} - T_m$ |
| Subcooling | $i = 3$<br>$k_3 = 6$<br>$Q_3 = C_s(T_{fm} - T_c)V$<br>$\mathrm{Bi}_3 = \mathrm{Bi}_s$<br>$\Delta T_{m3} = \dfrac{(T_{fm} - T_m) - (T_o - T_m)}{\ln\left[\dfrac{T_{fm} - T_m}{T_o - T_m}\right]}$ |

*Notes*: $A_s$ is the area through which heat is transferred
$\mathrm{Bi}_l$ is the Biot number for the unfrozen phase
$\mathrm{Bi}_s$ is the Biot number for the frozen phase
$Q_1$, $Q_2$, and $Q_3$ are the heats of precooling, phase change and subcooling, respectively
$\Delta T_{m1}$, $\Delta T_{m2}$, and $\Delta T_{m3}$ are the corresponding log-mean temperature driving forces
$T_c$ is the final thermal center temperature
$T_{fm}$ is the mean freezing point, assumed to be 1.5 K below the initial freezing point
$T_o$ is the mean final temperature
$V$ is the volume of the food item

where $C_l$ and $C_s$ are volumetric specific heats above and below freezing, respectively, $T_i$ is the initial food temperature, $L_f$ is the volumetric latent heat of freezing, and $V$ is the volume of the food item. By curve-fitting to existing experimental data, Pham (1986) proposed the following equation to determine the mean freezing temperature $T_{fm}$ for use in Equations (25) and (26):

$$T_{fm} = 1.8 + 0.26T_c + 0.105T_m \tag{27}$$

where $T_c$ is the final center temperature and $T_m$ is the freezing medium temperature.

Ilicali and Saglam (1987) and Ilicali et al. (1992) describe the development of a freezing time estimation method in which the freezing time is calculated as the sum of a cooling period and a freezing period. The cooling period is the time required for the food item to cool from an assumed uniform temperature distribution to a temperature distribution such that the food item's mass average temperature $\bar{T}$ equals its initial freezing temperature. The freezing period is the additional time required to reduce the center temperature of the food item to the final center temperature $T_c$.

In this method, cooling time for an infinite slab is determined from the following equation:

$$\frac{\bar{T}-T_m}{T_i-T_m} = A_1\frac{\sin\lambda_1}{\lambda_1}e^{-\lambda_1^2\mathrm{Fo}} \tag{28}$$

where $A_1$ is a parameter which depends on Biot number and is tabulated for infinite slabs by Kutateladze and Borishanskii (1966) and $\lambda_1$ is the first root of the following equation:

$$\cot\lambda_n = \lambda_n/\mathrm{Bi} \tag{29}$$

In Equations (28) and (29) $\bar{T}$ is the mass average temperature of the food item at the end of the cooling process, which is assumed to be the initial freezing point of the food, $T_m$ is the freezing medium temperature, $T_i$ is the initial temperature of the food item, Fo is the Fourier number for the cooling process (Fo = $\alpha_l\, t/L^2$), $\alpha_l$ is the thermal diffusivity of the unfrozen food, $t$ is the cooling time, $L$ is the half thickness of the infinite slab, and Bi is the Biot number for the cooling process (Bi = $hL/k_l$).

The time required to freeze the infinite slab to the final state with specified center temperature, $T_c$, can be calculated from the following expression:

$$\frac{T_c-T_m}{\bar{T}-T_m} = A_1e^{-\lambda_1^2\mathrm{Fo}} \tag{30}$$

where $A_1$ and $\lambda_1$ are as defined above, Bi is the Biot number for the freezing process (Bi = $hL/k_s$), Fo is the Fourier number for the freezing process, Fo = $\alpha_{eff}\, t/L^2$, and $\alpha_{eff}$ is the "effective" thermal diffusivity of the frozen food.

Because the cooling process is assumed to terminate when the mass average temperature of the food item reaches the initial freezing point, partial freezing occurs at locations within the food where the temperature is below the initial freezing point. To account for this partial freezing, an effective thermal diffusivity for the freezing process $\alpha_{eff}$ is defined as follows:

$$\alpha_{eff} = \frac{k_s}{\rho_s c_{eff}} \tag{31}$$

where $c_{eff}$ is the effective specific heat for the freezing process:

$$c_{eff} = \frac{\Delta H_{eff}}{\bar{T}-T_C} \tag{32}$$

and $\Delta H_{eff}$ is the effective enthalpy change for the freezing process, which, for an infinite slab, is assumed to be 75% of the experimentally determined enthalpy change necessary to reduce the temperature of the food item from an initial uniform freezing point temperature to a final state where the specified center temperature $T_c$ is obtained.

Using the method of Ilicali and Saglam (1987) and Ilicali et al. (1992), the cooling time of infinite cylinders can be determined from:

$$\frac{\bar{T} - T_m}{T_i - T_m} = A_1 \left[ \frac{2J_1(\lambda_1)}{\lambda_1} \right] e^{-\lambda_1^2 \mathrm{Fo}} \tag{33}$$

where $A_1$ is a parameter that depends on Biot number and is tabulated for infinite cylinders by Kutateladze and Borishanskii (1966), Fo is the Fourier number for the cooling process (Fo = $\alpha_l\, t/L^2$), and $\lambda_1$ is the first root of the following equation:

$$\frac{J_0(\lambda_n)}{J_1(\lambda_n)} = \frac{\lambda_n}{\mathrm{Bi}} \tag{34}$$

In Equations (33) and (34), $J_0$ and $J_1$ are Bessel functions of the first kind, order zero and one, respectively. The freezing time may be estimated from:

$$\frac{T_c - T_m}{\bar{T} - T_m} = A_1 e^{-\lambda_1^2 \mathrm{Fo}} \tag{35}$$

where Fo is the Fourier number for the freezing process (Fo = $\alpha_{eff}\, t/L^2$) and $\alpha_{eff}$ is calculated from Equations (31) and (32) assuming that the effective enthalpy difference is 73% of the experimentally determined enthalpy change necessary to reduce the temperature of the food item from an initial uniform freezing point temperature to a final state where the specified center temperature $T_c$ is obtained.

For a sphere, the cooling time can be determined from:

$$\frac{\bar{T} - T_m}{T_i - T_m} = 3A_1 \frac{\sin\lambda_1 - \lambda_1 \cos\lambda_1}{\lambda_1^3} e^{-\lambda_1^2 \mathrm{Fo}} \tag{36}$$

where $A_1$ is a parameter that depends on Biot number and is tabulated for spheres by Kutateladze and Borishanskii (1966), Fo is the Fourier number for the cooling process (Fo = $\alpha_l t/L^2$), and $\lambda_1$ is the first root of the following equation:

$$\tan\lambda_n = \frac{-\lambda_n}{\mathrm{Bi} - 1} \tag{37}$$

The freezing time of a sphere may be estimated from:

$$\frac{T_c - T_m}{\bar{T} - T_m} = A_1 e^{-\lambda_1^2 \mathrm{Fo}} \tag{38}$$

where Fo is the Fourier number for the freezing process (Fo = $\alpha_{eff}\, t/L^2$) and $\alpha_{eff}$ is calculated from Equations (31) and (32) assuming that the effective enthalpy difference is 70% of the experimentally determined enthalpy change necessary to reduce the temperature of the food item from an initial uniform freezing point temperature to a final state where the specified center temperature $T_c$ is obtained.

## Empirical Methods

Several empirical methods have been developed for estimating food freezing times (Albin et al. 1979, Bazan and Mascheroni 1984, Hayakawa et al. 1983, Salvadori and Mascheroni 1991, Sheen and Hayakawa 1991). Many of these methods are limited to only a specific food geometry or are cumbersome to use. An exception to this statement is the method of Salvadori and Mascheroni (1991).

Salvadori and Mascheroni (1991) suggest that the temperature at the thermal center of a food item can be related to a dimensionless freezing time parameter *X*, which accounts for the effects of time, process parameters, thermophysical properties, and product size. The dimensionless freezing time parameter is given as follows:

$$X = \frac{\mathrm{Fo}\left[\frac{T_m - T_f}{T_f}\right]^m}{\left(\frac{1}{\mathrm{Bi}} + b\right)\left[\frac{T_f - T_i}{T_f}\right]^n} \tag{39}$$

where Fo is the Fourier number ($\mathrm{Fo} = \alpha_l t/L^2$), $\alpha_l$ is the thermal diffusivity of the unfrozen food, $t$ is the freezing time, $L$ is the half thickness of an infinite slab or the radius of an infinite cylinder or sphere, Bi is the Biot number ($\mathrm{Bi} = hL/k_l$), and $k_l$ is the thermal conductivity of the unfrozen food. The experimentally determined constants $m$, $n$, and $b$, given in Table 6, depend on product geometry.

Salvadori and Mascheroni determined that the thermal center temperature of a food item $T_c$ and its dimensionless freezing time parameter $X$ are linearly related as follows:

$$X = AT_c + B \tag{40}$$

The constants $A$ and $B$ depend on geometry and are given in Table 6. Equations (39) and (40) can be combined and rearranged to give the freezing time as follows:

$$t = \frac{L^2(AT_c + B)\left(\frac{1}{\mathrm{Bi}} + b\right)\left[\frac{T_f - T_i}{T_f}\right]^n}{\alpha_l\left[\frac{T_m - T_f}{T_f}\right]^m} \tag{41}$$

Salvadori and Mascheroni state that the above equation is valid for $-18°\mathrm{C} \le T_c \le -5°\mathrm{C}$.

**Table 6. Parameters used in Freezing Time Estimation Method of Salvadori and Mascheroni (1991)**

| Geometry | ***m*** | ***n*** | ***b*** | ***A*** | ***B*** |
|---|---|---|---|---|---|
| Slab[a] | 1.04 | 0.09 | 0.18 | −1.08125 | 62.9375 |
| Slab[b] | 1.03 | 0.10 | 0.16 | −0.94250 | 62.4350 |
| Infinite cylinder | 1.00 | 0.09 | 0.17 | −0.46875 | 28.7625 |
| Sphere | 0.90 | 0.06 | 0.18 | −0.16875 | 15.3625 |

[a]Heat transfer perpendicular to fibers. [b]Heat transfer parallel to fibers.

## PERFORMANCE OF FOOD FREEZING TIME ESTIMATION METHODS

The performance of each of the previously discussed food freezing time estimation methods was analyzed by comparing calculated freezing times with empirical freezing time data available from the literature. The empirical freezing time data set is shown in Table 7.

**Table 7. Empirical Freezing Time Data Set**

| Shape | Number of Data Points | Material | Reference |
|---|---|---|---|
| Infinite slab | 43 | Tylose gel | Cleland and Earle (1977) |
| Infinite slab | 6 | Mashed potato | Cleland and Earle (1977) |
| Infinite slab | 6 | Minced lean beef | Cleland and Earle (1977) |
| Infinite slab | 23 | Tylose gel | Hung and Thompson (1983) |
| Infinite slab | 9 | Lean beef | Hung and Thompson (1983) |
| Infinite slab | 9 | Mashed potato | Hung and Thompson (1983) |
| Infinite slab | 9 | Carp meat | Hung and Thompson (1983) |
| Infinite slab | 9 | Ground beef | Hung and Thompson (1983) |
| Infinite slab | 32 | Tylose gel | Pham and Willix (1990) |
| Infinite cylinder | 30 | Tylose gel | Cleland and Earle (1979) |
| Sphere | 30 | Tylose gel | Cleland and Earle (1979) |
| Sphere | 20 | Apple | Ilicali and Saglam (1987) |
| Sphere | 48 | Beef | Tocci and Mascheroni (1994) |

**Table 8. Thermal Property Data Used for Calculation of Freezing Times**

| Property | | Tylose Gel[a] | Mashed Potato[a] | Lean Beef[a] | Ground Beef[a] | Carp[a] | Apples[b] |
|---|---|---|---|---|---|---|---|
| $k_l$ | W/(m·K) | 0.55 | 0.53 | 0.50 | 0.44 | 0.48 | 0.43 |
| $k_s$ | W/(m·K) | 1.65 | 1.90 | 1.55 | 1.45 | 1.65 | 1.45 |
| $C_l$ | μJ/(m³·K) | 3.71 | 3.66 | 3.65 | 3.38 | 3.70 | 3.12 |
| $C_s$ | μJ/(m³·K) | 1.90 | 1.95 | 1.90 | 1.95 | 2.10 | 1.54 |
| $L_f$ | μJ/m³ | 209 | 235 | 209 | 188 | 218 | 230 |
| $T_f$ | °C | −0.6 | −0.6 | −1.0 | −1.2 | −0.8 | −1.0 |

[a]Cleland and Earle (1984) [b]Ramaswamy and Tung (1981)

**Table 9. Effective Thermal Diffusivities[a]**

| Shape | Material | $\alpha_{eff}$, mm²/s[b] | $\alpha_{eff}$, mm²/s[c] |
|---|---|---|---|
| Infinite slab | Tylose gel | 0.083 | 0.151 |
| | Mashed potato | 0.094 | 0.159 |
| | Carp | 0.085 | 0.142 |
| | Lean beef | 0.082 | 0.137 |
| | Ground beef | 0.079 | 0.134 |
| Infinite cylinder | Tylose gel | 0.085 | — |
| Sphere | Tylose gel | 0.089 | — |
| | Apple | 0.097 | 0.159 |
| | Ground beef | — | 0.134 |

[a]From Ilicali and Saglam (1987)
[b]For final center temperature of −10°C
[c]For final center temperature of −18°C

Some of the experimental data shown in Table 7 was generated by freezing tylose gel. This material, first introduced by Riedel (1960), is a commonly used food analog consisting of 23% methylcellulose and 77% water. Its thermal properties are similar to those of lean beef and its freezing behavior closely resembles that of foods with high water content (Pham and Willix 1990). The thermal properties of tylose gel, as well as the thermal properties of the other food items used in this study, are given in Table 8. In addition, the effective thermal diffusivities of the food items are given in Table 9.

**Table 10. Statistical Analysis of Food Freezing Time Estimation Methods Applicable to Infinite Slabs**

| Estimation Method | Ave. Absolute Prediction Error, % | Standard Deviation, % | 95% Confidence Range, % | Kurtosis | Skewness |
|---|---|---|---|---|---|
| Cleland and Earle (1977) | 5.62 | 5.00 | ±0.818 | 1.96 | 1.39 |
| Hung and Thompson (1983) | 6.66 | 7.12 | ±1.16 | 5.79 | 2.23 |
| Pham (1984) | 5.85 | 4.65 | ±0.761 | 2.03 | 1.37 |
| Pham (1986) | 6.56 | 5.02 | ±0.821 | 1.72 | 1.24 |
| Ilicali and Saglam (1987) | 12.9 | 15.9 | ±2.60 | 6.16 | 2.47 |
| Lacroix and Castaigne (1987) | 7.38 | 6.77 | ±1.11 | 1.49 | 1.37 |
| Salvadori and Mascheroni (1991) | 7.32 | 5.56 | ±0.909 | 0.674 | 0.943 |

## Performance of Food Freezing Time Estimation Methods Applicable to Infinite Slabs

Table 10 summarizes the statistical analysis which was performed on the freezing time estimation methods applicable to infinite slabs of food. Because the freezing time estimation method of Cleland and Earle (1977) was based on a curve fit to their data, this method performs well when compared against their data for tylose gel, mashed potatoes, and minced lean beef. The method of Cleland and Earle (1977) had an average absolute prediction error of 2.16% when it was used to predict freezing times from their data set. Because the final center temperature for the Hung and Thompson (1983) data set and the Pham and Willix (1990) data set was –18°C, the final center temperature correction factor was required by the Cleland and Earle (1977) method for these two data sets. Without the correction factor, the Cleland and Earle method produced an average absolute prediction error of 12.3% for both the Hung and Thompson data set and the Pham and Willix data set. However, with the final center temperature correction factor applied, the average absolute prediction error was reduced to 7.52% for both the Hung and Thompson (1983) and Pham and Willix (1990) data sets. The average absolute prediction error of the Cleland and Earle method for all tests combined was 5.62%. The distribution of the absolute prediction errors was fairly well peaked around the mean, and the 95% confidence range was among the lowest (±0.818%).

Likewise, the food freezing time estimation method of Hung and Thompson (1983) was based on a curve fit to their data, and thus, their method performs well when compared with their data for tylose gel, lean beef, mashed potatoes, carp and ground beef. When comparing the results of the Hung and Thompson method to the freezing times from the data set of Cleland and Earle (1977), the final center temperature correction factor reduced the average absolute prediction error from 23.6% to 11.3%. Overall, the Hung and Thompson (1983) method yielded an average absolute prediction error of 6.66%, with a 95% confidence range of ±1.16%. The absolute prediction error was fairly well distributed about the mean.

The two estimation methods developed by Pham (1984, 1986) performed consistently when compared against all of the experimental data sets. The average absolute prediction error for the Pham (1984) method was 5.85% with a 95% confidence range of ±0.761%, while the average absolute prediction error for the slightly simpler Pham (1986) method was 6.56% with a 95% confidence range of ±0.821%. The distribution of absolute prediction errors for both of Pham's methods was relatively flat.

The food freezing time estimation method of Ilicali and Saglam (1987) performed satisfactorily, achieving an average absolute prediction error of 7.77%, when compared against the data set of Cleland and Earle (1977) and Pham and Willix (1990). However, when compared to the Hung and Thompson (1983) data set, the Ilicali and Saglam method produced an average absolute error of 20.9%. Overall, the average absolute prediction error of the Ilicali and Saglam method was found to be 12.9% with a large 95% confidence range of ±2.60%. Ilicali and Saglam (1987) noted that large prediction errors occurred when the freezing period temperature ratio $(T_f - T_m)/(\bar{T} - T_m)$ was less than 0.3. They found that by subdividing the freezing period into a primary freezing period and a secondary freezing period, these large absolute prediction errors could be reduced.

Overall, the method of Lacroix and Castaigne (1987) produced an average absolute prediction error of 7.38% with a 95% confidence range of ±1.11%. Their method performed best when compared to the data sets of Cleland and Earle (1977) and Pham and Willix (1990), resulting in an average absolute prediction error of 3.69%. The Lacroix and Castaigne method performed its worst when compared to the data set of Hung and Thompson (1983), producing an average absolute prediction error of 12.4%.

The food freezing time estimation method of Salvadori and Mascheroni (1991) performed satisfactorily overall, achieving an average absolute prediction error of 7.32% with a modest 95% confidence range of ±0.909%. Its best results were obtained when compared to the data sets of Cleland and Earle (1977) and Pham and Willix (1990). The average absolute error of the Salvadori and Mascheroni method for these two data sets was found to be 6.13%. The Salvadori and Mascheroni method produced an average absolute error of 9.06% when compared to the data set of Hung and Thompson (1983).

## Performance of Food Freezing Time Estimation Methods Applicable to Infinite Cylinders

The statistical analysis of the freezing time estimation methods applicable to infinite cylinders of food is given in Table 11. As with infinite slabs, the method of Cleland and Earle (1979) performs well when compared against their data for tylose gel cylinders. An average absolute prediction error of 2.35% was obtained when used to predict freezing times from their data set.

**Table 11. Statistical Analysis of Food Freezing Time Estimation Methods Applicable to Infinite Cylinders**

| Estimation Method | Ave. Absolute Prediction Error, % | Standard Deviation, % | 95% Confidence Range, % | Kurtosis | Skewness |
|---|---|---|---|---|---|
| Cleland and Earle (1979) | 2.35 | 1.74 | ±0.649 | 0.530 | 1.08 |
| Pham (1984) | 4.44 | 2.92 | ±1.09 | −0.731 | 0.421 |
| Pham (1986) | 3.93 | 2.76 | ±1.03 | 0.077 | 0.640 |
| Ilicali and Saglam (1987) | 3.44 | 3.13 | ±1.17 | −0.244 | 0.973 |
| Lacroix and Castaigne (1987) | 3.65 | 3.36 | ±1.25 | 0.898 | 1.20 |
| Salvadori and Mascheroni (1991) | 7.32 | 3.57 | ±1.33 | −1.09 | 0.00 |

The food freezing time estimation methods developed by Pham (1984, 1986), Ilicali and Saglam (1987) and Lacroix and Castaigne (1987) performed equally well, each having an average absolute prediction error of less than 4.44% and a 95% confidence range of less than ±1.25%. The method of Salvadori and Mascheroni (1991) produced a large average absolute prediction error of 7.32% with a large 95% confidence range of ±1.33%.

### Performance of Food Freezing Time Estimation Methods Applicable to Spheres

The statistical analysis of the freezing time estimation methods applicable to spherical food items is given in Table 12. The method of Cleland and Earle (1979) performs well when compared against their data for tylose gel spheres. An average absolute prediction error of 3.29% was obtained by the method of Cleland and Earle (1979) when used to predict freezing times from their data set. The average absolute prediction error of the Cleland and Earle method when compared to all spherical food data was 9.92% with a 95% confidence range of ±1.99%.

**Table 12. Statistical Analysis of Food Freezing Time Estimation Methods Applicable to Spheres**

| Estimation Method | Ave. Absolute Prediction Error, % | Standard Deviation, % | 95% Confidence Range, % | Kurtosis | Skewness |
|---|---|---|---|---|---|
| Cleland and Earle (1979) | 9.92 | 9.94 | ±1.99 | 2.24 | 1.60 |
| Pham (1984) | 12.3 | 12.5 | ±2.50 | 1.62 | 1.59 |
| Pham (1986) | 10.4 | 12.4 | ±2.49 | 2.89 | 1.88 |
| Ilicali and Saglam (1987) | 6.85 | 4.75 | ±0.952 | 0.680 | 0.930 |
| Lacroix and Castaigne (1987) | 11.0 | 12.4 | ±2.49 | 3.83 | 2.01 |
| Salvadori and Mascheroni (1991) | 7.53 | 6.85 | ±1.37 | 3.25 | 1.72 |

Both of the methods developed by Pham (1984, 1986) accurately predicted the freezing times of both the Cleland and Earle (1979) data set and the Tocci and Mascheroni (1994) data set, achieving an average absolute prediction error of less than 7.06%. Comparison of both of Pham's methods with the Ilicali and Saglam (1987) data set for apples, however, produced an average absolute prediction error of 34.3%. Overall, the average absolute prediction error was less than 12.3% with a 95% confidence range less than ±2.50% for both of Pham's methods.

The method of Ilicali and Saglam (1987) yielded its best results when compared to their data set on apples. When compared to this data set, an average absolute prediction error of 6.39% was obtained. For all spherical data, Ilicali and Saglam's method produced the lowest average absolute prediction error, 6.85%, also the narrowest 95% confidence range, ±0.952%.

The method of Lacroix and Castaigne (1987a, 1987b, 1988) performed similarly to that of Pham's (1984, 1986) methods. Overall, the Lacroix and Castaigne method yielded an average absolute prediction error of 11.0% with a large 95% confidence range of ±2.49%. The method of Salvadori and Mascheroni (1991) performed well overall, yielding an average absolute prediction error of 7.53% with a fairly narrow 95% confidence range of ±1.37%.

## CONCLUSIONS

The food freezing time estimation methods developed by Cleland and Earle (1977, 1979) performed well for infinite slabs and infinite cylinders. The methods of Pham (1984, 1986) performed better with infinite slabs and infinite cylinders of food than they did for spherical food items. The method of Ilicali and Saglam (1987) produced low prediction errors for cylindrical and spherical food items, but produced large prediction errors for infinite slabs of food. The method of Lacroix and Castaigne (1987a, 1987b, 1988) performed best for infinite cylinders of food and produced high prediction errors for infinite slabs and spheres. The method of Salvadori and Mascheroni (1991) performed consistently with all three regular shapes, producing moderately large prediction errors.

In summary, for infinite slabs, the methods of Pham (1984, 1986), Hung and Thompson (1983) and Cleland and Earle (1977) all performed equally well. For infinite cylinders, the methods of Pham (1986) and Cleland and Earle (1979) performed the best while the methods of Ilicali and Saglam (1987) and Lacroix and Castaigne (1987) also did well. Finally, for spheres, the methods of Ilicali and Saglam (1987) and Salvadori and Mascheroni (1991) gave the best results.

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## NOMENCLATURE

| | |
|---|---|
| $A$ | parameter given in Table 6 |
| $A_1$ | parameter in Equations (28),(30),(33),(35),(36),(38) |
| $A_s$ | surface area of food item, $m^2$ |
| $b$ | parameter given in Table 6 |
| $B$ | parameter given in Table 6 |
| Bi | Biot number |
| $Bi_1$ | Biot number for precooling; $Bi_1 = (Bi_l + Bi_s)/2$ |
| $Bi_2$ | Biot number for phase change; $Bi_2 = Bi_s$ |
| $Bi_3$ | Biot number for subcooling; $Bi_3 = Bi_s$ |
| $Bi_c$ | Biot number evaluated at $k_c$; $Bi_c = hL/k_c$ |
| $Bi_l$ | Biot number for unfrozen food; $Bi_l = hD/k_l$ |
| $Bi_s$ | Biot number for fully frozen food; $Bi_s = hD/k_s$ |
| $c$ | specific heat of food item, J/(kg·K) |
| $c_{eff}$ | effective specific heat of food item, J/(kg·K) |
| $C_l$ | volumetric specific heat of unfrozen food, J/(m$^3$·K) |
| $C_s$ | volumetric specific heat of fully frozen food, J/(m$^3$·K) |
| $D$ | slab thickness or cylinder/sphere diameter, m |
| $f$ | cooling time parameter |
| $f_1$ | cooling time parameter for precooling |
| $f_3$ | cooling time parameter for subcooling |
| Fo | Fourier number; Fo = $\alpha t/L$ |
| $h$ | heat transfer coefficient, W/(m$^2$·K) |
| $j$ | cooling time parameter |
| $j_1$ | cooling time parameter for precooling |
| $j_3$ | cooling time parameter for subcooling |
| $j_c$ | cooling time parameter applicable to thermal center |
| $J_0(x)$ | Bessel function of the first kind, order zero |
| $J_1(x)$ | Bessel function of the first kind, order one |
| $k$ | thermal conductivity of food item, W/(m·K) |
| $k_c$ | thermal conductivity of food evaluated at $(T_f + T_m)/2$, W/(m·K) |
| $k_l$ | thermal conductivity of unfrozen food, W/(m·K) |
| $k_s$ | thermal conductivity of fully frozen food, W/(m·K) |
| $L$ | half thickness of slab or radius of cylinder/sphere, m |
| $L_f$ | volumetric latent heat of fusion, J/m$^3$ |
| $m$ | parameter given in Table 6 |
| $n$ | parameter given in Table 6 |
| $P$ | Plank's geometry factor |
| Pk | Plank number; $C_l (T_i - T_f)/\Delta H$ |
| $Q_1$ | volumetric heat of precooling, J/m$^3$ |
| $Q_2$ | volumetric heat of phase change, J/m$^3$ |
| $Q_3$ | volumetric heat of subcooling, J/m$^3$ |
| $R$ | Plank's geometry factor |
| Ste | Stefan number; $C_s(T_f - T_m)/\Delta H$ |
| $t$ | cooling or freezing time, s |
| $t_1$ | precooling time, s |
| $t_2$ | phase change time, s |
| $t_3$ | subcooling time, s |
| $T$ | product temperature, °C |
| $T_c$ | final center temperature of food item, °C |
| $T_f$ | initial freezing temperature of food item, °C |
| $T_{fm}$ | mean freezing temperature, °C |
| $T_i$ | initial temperature of food item, °C |
| $T_m$ | freezing medium temperature, °C |
| $T_o$ | mean final temperature, °C |

$T_{ref}$ reference temperature for freezing time correction factor, °C
$\bar{T}$ average temperature of food item, °C
$u$ parameter given in Table 1
$U$ parameter in Equations (9) and (10); $U = \Delta T/(T_f - T_m)$
$v$ parameter given in Table 2
$V$ volume of food item, $m^3$
$w$ parameter given in Table 3
$x$ coordinate direction
$X$ dimensionless freezing time parameter given by Equation (39)
$y$ coordinate direction
$z$ coordinate direction
$\alpha$ thermal diffusivity of food, $m^2/s$
$\alpha_{eff}$ effective thermal diffusivity of food, $m^2/s$
$\alpha_l$ thermal diffusivity of unfrozen food, $m^2/s$
$\Delta H$ volumetric enthalpy difference, $J/m^3$
$\Delta H_1$ volumetric enthalpy difference, $J/m^3$; $\Delta H_1 = C_l(T_i - T_{fm})$
$\Delta H_2$ volumetric enthalpy difference, $J/m^3$; $\Delta H_2 = L_f + C_s(T_{fm} - T_c)$
$\Delta H_{10}$ volumetric enthalpy difference between the initial freezing temperature $T_f$ and −10°C, $J/m^3$
$\Delta H_{18}$ volumetric enthalpy difference between the initial temperature $T_i$ and −18°C, $J/m^3$
$\Delta H_{eff}$ effective enthalpy difference, J/kg
$\Delta T$ weighted average temperature difference (°C) given by Equation (8)
$\Delta T_1$ temperature difference, °C; $\Delta T_1 = (T_i + T_{fm})/2 - T_m$
$\Delta T_2$ temperature difference, °C; $\Delta T_2 = T_{fm} - T_m$
$\Delta T_{m1}$ temperature difference for precooling, °C
$\Delta T_{m2}$ temperature difference for phase change, °C
$\Delta T_{m3}$ temperature difference for subcooling, °C
$\lambda_1$ first root of transcendental equation
$\rho$ density of food item, $kg/m^3$
$\rho_s$ density of fully frozen food item, $kg/m^3$

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