FREEZING TIMES OF REGULARLY SHAPED FOOD ITEMS

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ABSTRACT

The freezing of food is one of the most significant applications of refrigeration. In order for freezing operations to be cost-effective, it is necessary to optimally design the refrigeration equipment. This requires estimation of the freezing times of foods. Numerous semi-analytical/empirical methods for predicting food freezing times have been proposed. The designer of food refrigeration facilities is thus faced with the challenge of selecting an appropriate estimation method from the plethora of available methods. Therefore, a review of selected semi-analytical/empirical food freezing time prediction methods applicable to regularly shaped food items is given in this paper. The performance of these various methods is evaluated by comparing their results to experimental freezing time data obtained from the literature.

Introduction

Preservation of food is one of the most significant applications of refrigeration. It is known that the freezing of food effectively reduces the activity of micro-organisms and enzymes, thus retarding deterioration. In addition, crystallization of water reduces the amount of liquid water in food items and inhibits microbial growth [1].

In order for food freezing operations to be cost-effective, it is necessary to optimally design the refrigeration equipment to fit the specific requirements of the particular freezing application. The design of such refrigeration equipment requires estimation of the freezing times of foods, as well as the corresponding refrigeration loads.

Numerous methods for predicting food freezing times have been proposed. The designer is thus faced with the challenge of selecting an appropriate estimation method from the plethora of available methods. This paper focuses upon those methods which are applicable to regularly shaped food items. The performance of these various methods is evaluated by comparing their results to experimental freezing time data obtained from the literature.

Thermodynamics of the Freezing Process

The freezing of food is a complex process. Prior to freezing, sensible heat must be removed from the food to decrease its temperature from the initial temperature to the initial freezing point of the food. This initial freezing point is somewhat lower than the freezing point of pure water due to dissolved substances in the moisture within the food. At the initial freezing point, a portion of the water within the food crystallizes and the remaining solution becomes more concentrated. Thus, the freezing point of the unfrozen portion of the food is further reduced. As the temperature continues to decrease, the formation of ice crystals increases the concentration of the solutes in solution and depresses the freezing point further. Thus, it is evident that during the freezing process, the ice and water fractions in the frozen food depend upon temperature. Since the thermophysical properties of ice and liquid water are quite different, the corresponding properties of the frozen food are temperature dependent. Therefore, due to these complexities, it is not possible to derive exact analytical solutions for the freezing times of foods.

Numerical estimates of food freezing times can be obtained using appropriate finite element or finite difference computer programs. However, the effort required to perform this task makes it impractical for the design engineer. In addition, two-dimensional and three-dimensional simulations require time consuming data preparation and significant computing time. Hence, the majority of the research effort to date has been in the development of semi-analytical/empirical food freezing time prediction methods which make use of simplifying assumptions.

Freezing Time Estimation Methods

In the following discussion, the basic freezing time estimation method developed by Plank is discussed first, followed by a discussion of those methods which are based upon modifications of Plank's equation. The discussion then focuses upon those methods in which the freezing time is calculated as the sum of the precooling, phase change and subcooling times. The next section deals with empirical freezing time estimation methods. Finally, the performance of each of the described freezing time estimation methods is evaluated by comparison to experimental freezing time data found in the literature.

Plank's Equation

The most widely known basic method for estimating the freezing times of foods is that developed by Plank [2,3]. In this method, it is assumed that only convective heat transfer occurs between the food item and the surrounding cooling medium. In addition, it is assumed that the temperature of the food item is its initial freezing temperature and that this temperature is constant throughout the freezing process. Furthermore, a constant thermal conductivity for the frozen region is assumed. Plank's freezing time estimation method is given as follows:

$$t = \frac{L_f}{T_f - T_m}\left[\frac{PD}{h} + \frac{RD^2}{k_s}\right] \quad (1)$$

where *P* and *R* are geometric factors. For the infinite slab, *P* = ½ and *R* = 1/8. For a sphere, *P* and *R* are 1/6 and 1/24, respectively, and for an infinite cylinder, *P* = 1/4 and *R* = 1/16.

The geometric factors, *P* and *R*, provide insight as to the effect of shape upon freezing time. Plank's shape factors indicate that an infinite slab of thickness *D*, an infinite cylinder of diameter *D* and a sphere of diameter *D*, if exposed to the same conditions, would have freezing times in the ratio of 6:3:2. Hence, a cylinder will freeze in half the time of a slab and a sphere will freeze in one-third the time of a slab.

Various researchers have noted that Plank's method does not accurately predict the freezing times of foods. This is due, in part, to the fact that Plank's method assumes that food freezing takes place at a constant temperature, and not over a range of temperatures as is the case in actual food freezing processes. In addition, the thermal conductivity of the frozen food is assumed to be constant, but in reality, the thermal conductivity varies greatly during freezing. Another limitation of Plank's equation is that it neglects the removal of sensible heat above the freezing point. However, Plank's method does have the advantage of being a simple model for predicting food freezing time. Subsequently, researchers have focused upon development of improved semi-analytical/empirical cooling and freezing time estimation methods which account for precooling and subcooling times, non-constant thermal properties, and phase change over a range of temperatures.

Modifications to Plank's Equation

Cleland and Earle [4,5] improved upon Plank's model by incorporating corrections to account for the removal of sensible heat both above and below the initial freezing point of the food as well as temperature variation during freezing. Regression equations were developed to estimate the geometric parameters, *P* and *R*, for infinite slabs, infinite cylinders and spheres. In these regression equations, the effects of surface heat transfer, precooling and final subcooling are accounted for by means of the Biot number, *Bi*, the Plank number, *Pk*, and the Stefan number, *Ste*, respectively. The latent heat, L_f, in Plank's equation is replaced with the volumetric enthalpy change of the food, $?H_{10}$, between the freezing temperature, T_f, and the final center temperature, assumed to be -10°C.

Hung and Thompson [6] also improved upon Plank's equation to develop an alternative freezing time estimation method for infinite slabs. Their equation incorporates the volumetric change in enthalpy, $?H_{18}$, for the freezing process as well as a weighted average temperature difference between the initial temperature of the food and the freezing medium temperature. Empirical equations were developed to estimate the geometric factors, *P* and *R*.

Precooling, Phase Change and Subcooling Time Calculations

Numerous researchers have taken a different approach to account for the effects of sensible heat removal above and below the initial freezing point. In these methods, the total freezing time, t, is the sum of the precooling, phase change and subcooling times:

$$t = t_1 + t_2 + t_3 \qquad (2)$$

where t_1, t_2 and t_3 are the precooling, phase change and subcooling times, respectively.

Lacroix and Castaigne [7-9] suggested the use of f and j factors, based upon the slope and intercept of the food temperature versus time curve, to determine the precooling and subcooling times of foods. Lacroix and Castaigne gave the following expression for estimating the precooling time, t_1:

$$t_1 = f_1 \log\left[j_1 \frac{T_m - T_i}{T_m - T_f} \right] \qquad (3)$$

A similar expression is given for estimating the subcooling time, t_3:

$$t_3 = f_3 \log\left[j_3 \frac{T_m - T_f}{T_m - T_c} \right] \qquad (4)$$

Lacroix and Castaigne [7-9] model the phase change time, t_2, with Plank's equation. They noted that by making adjustments to Plank's geometric factors, P and R, better agreement between predicted freezing times and experimental data was obtained. Using regression analysis, Lacroix and Castaigne suggested that the geometric factors should be as follows. For infinite slabs: P = 0.51233 and R = 0.15396; for infinite cylinders: P = 0.27553 and R = 0.07212; and, for spheres: P = 0.19665 and R = 0.03939.

Pham [10] also devised a food freezing time estimation method, similar to Plank's equation, in which sensible heat effects are considered by calculating precooling, phase change and subcooling times separately. In addition, Pham suggested the use of a mean freezing point, which is assumed to be 1.5 K below the initial freezing point of the food, to account for freezing which takes place over a range of temperatures. Pham's freezing time estimation method is stated in terms of the volume and surface area of the food item and is therefore applicable to food items of any shape. Pham [11] subsequently simplified the previous freezing time estimation method [10] to yield a single equation which includes precooling, phase change and subcooling.

Ilicali and Saglam [12] and Ilicali et al. [13] describe the development of a freezing time estimation method in which the freezing time is calculated as the sum of a cooling period and a freezing period. The cooling period is the time required for the food item to cool from an assumed uniform temperature distribution to a temperature distribution such that the food item's mass average temperature, _, is equal to its initial freezing temperature. The freezing period is the additional time required to reduce the center temperature of the food item to the final center temperature, T_c.

Empirical Methods

Several empirical methods have been developed for estimating food freezing times [14-18]. Many of these methods are limited to only a specific food geometry or are cumbersome to use. An exception to this statement is the method of Salvadori and Mascheroni [17].

Salvadori and Mascheroni [17] suggest that the temperature at the thermal center of a food item can be related to a dimensionless freezing time parameter, X, which accounts for the effects of time, process parameters, thermophysical properties and product size. Salvadori and Mascheroni determined that the thermal center temperature of a food item, T_c, and its dimensionless freezing time parameter, X, are linearly related, and thus, an expression for freezing time can be obtained.

Performance of Food Freezing Time Estimation Methods

The performance of each of the previously discussed food freezing time estimation methods was analyzed by comparing calculated freezing times with empirical freezing time data available from the literature [4-6,12,19,20]. The empirical freezing time data set consists of 274 freezing time data points for the following food items: 1) Apple, 2) Beef, 3) Carp meat, 4) Mashed potato, 5) Minced lean beef, and, 6) Tylose gel. Tylose gel, first introduced by Riedel [21], is a commonly used food analogue consisting of 23% methylcellulose and 77% water. Its thermal properties are similar to those of lean beef and its freezing behavior closely resembles that of foods with high water content [19].

Performance of Food Freezing Time Estimation Methods Applicable to Infinite Slabs

Table 1 summarizes the statistical analysis which was performed on the freezing time estimation methods applicable to infinite slabs of food. For each of the methods, the following information is given in Table 1: the average absolute prediction error (%), the standard deviation (%), the 95% confidence range (%), the kurtosis and the skewness.

Since the freezing time estimation method of Cleland and Earle [4] was based on a curve fit to their data, this method performs well when compared against their data for tylose gel, mashed potatoes and minced lean beef. The method of Cleland and Earle [4] had an average absolute prediction error of 2.16% when it was used to predict freezing times from their data set. The average absolute prediction error of the Cleland and Earle method for all tests combined was 5.62%. The distribution of the errors was fairly well peaked around the mean, and the 95% confidence range was among the lowest (±0.818%).

TABLE 1
Statistical Analysis of Food Freezing Time Estimation Methods Applicable to Infinite Slabs

Estimation Method	Average Absolute Prediction Error (%)	Standard Deviation (%)	95% Confidence Range (%)	Kurtosis	Skewness
Cleland and Earle [4]	5.62	5.00	±0.818	1.96	1.39
Hung and Thompson [6]	6.66	7.12	±1.16	5.79	2.23
Ilicali and Saglam [12]	12.9	15.9	±2.60	6.16	2.47
Lacroix and Castaigne [7-9]	7.38	6.77	±1.11	1.49	1.37
Pham [10]	5.85	4.65	±0.761	2.03	1.37

Pham [11]	6.56	5.02	±0.821	1.72	1.24
Salvadori and Mascheroni [17]	7.32	5.56	±0.909	0.674	0.943

Likewise, the food freezing time estimation method of Hung and Thompson [6] was based on a curve fit to their data, and thus, their method performs well when compared with their data for tylose gel, lean beef, mashed potatoes, carp and ground beef. Overall, the Hung and Thompson [6] method yielded an average absolute prediction error of 6.66%, with a 95% confidence range of ±1.16%. The prediction error was fairly well distributed about the mean.

The two estimation methods developed by Pham [10,11] performed consistently when compared against all of the experimental data sets. The overall average absolute prediction error for the Pham [10] method was 5.85% with a 95% confidence range of ±0.761%, while the overall average absolute prediction error for the slightly simpler Pham [11] method was 6.56% with a 95% confidence range of ±0.821%. The distribution of prediction errors for both of Pham's methods was relatively flat.

The food freezing time estimation method of Ilicali and Saglam [12] performed satisfactorily, achieving an average absolute prediction error of 7.77%, when compared against the data set of Cleland and Earle [4] and Pham and Willix [19]. However, when compared to the Hung and Thompson [6] data set, the Ilicali and Saglam method produced an average absolute prediction error of 20.9%. Overall, the average absolute prediction error of the Ilicali and Saglam method was found to be 12.9% with a large 95% confidence range of ±2.60%. Ilicali and Saglam [12] noted that large prediction errors occurred when the freezing period temperature ratio, $(T_f - T_m)/(\_ - T_m)$, was less than 0.3. They found that by subdividing the freezing period into a primary freezing period and a secondary freezing period, these large prediction errors could be reduced.

Overall, the method of Lacroix and Castaigne [7-9] produced an average absolute prediction error of 7.38% with a 95% confidence range of ±1.11%. Their method performed best when compared to the data sets of Cleland and Earle [4] and Pham and Willix [19], resulting in an average absolute prediction error of 3.69%. The Lacroix and Castaigne method performed its worst when compared to the data set of Hung and Thompson [6], producing an average absolute prediction error of 12.4%.

The food freezing time estimation method of Salvadori and Mascheroni [17] performed satisfactorily overall, achieving an average absolute prediction error of 7.32% with a modest 95% confidence range of ±0.909%. Its best results were obtained when compared to the data sets of Cleland and Earle [4] and Pham and Willix [19]. The average absolute prediction error of the Salvadori and Mascheroni method for these two data sets was found to be 6.13%. The Salvadori and Mascheroni method produced an average absolute prediction error of 9.06% when compared to the data set of Hung and Thompson [6].

Performance of Food Freezing Time Estimation Methods Applicable to Infinite Cylinders

The performance of the freezing time estimation methods applicable to infinite cylinders of food is given in Table 2, which shows the average absolute prediction error (%), the standard deviation (%), the 95% confidence range (%), the kurtosis and the skewness.

TABLE 2
Statistical Analysis of Food Freezing Time Estimation Methods Applicable to Infinite Cylinders

Estimation Method	Average Absolute Prediction Error (%)	Standard Deviation (%)	95% Confidence Range (%)	Kurtosis	Skewness
Cleland and Earle [5]	2.35	1.74	±0.649	0.530	1.08
Ilicali and Saglam [12]	3.44	3.13	±1.17	-0.244	0.973
Lacroix and Castaigne [7-9]	3.65	3.36	±1.25	0.898	1.20
Pham [10]	4.44	2.92	±1.09	-0.731	0.421
Pham [11]	3.93	2.76	±1.03	0.077	0.640
Salvadori and Mascheroni [17]	7.32	3.57	±1.33	-1.09	0.00

Since the method of Cleland and Earle [5] was based on a curve fit to their data, this method performs well when compared against their data for tylose gel cylinders. An average absolute prediction error of 2.35% was obtained by the method of Cleland and Earle [5] when used to predict freezing times from their data set.

The food freezing time estimation methods developed by Pham [10,11], Ilicali and Saglam [12] and Lacroix and Castaigne [7-9] performed equally well, each having an average absolute prediction error of less than 4.44% and a 95% confidence range of less than ±1.25%. The method of Salvadori and Mascheroni [17] produced a large average absolute prediction error of 7.32% with a large 95% confidence range of ±1.33%.

Performance of Food Freezing Time Estimation Methods Applicable to Spheres

The performance of the freezing time estimation methods applicable to spherical food items is given in Table 3, which shows the average absolute prediction error (%), the standard deviation (%), the 95% confidence range (%), the kurtosis and the skewness.

Since the method of Cleland and Earle [5] was based on a curve fit to their data, this method performs well when compared against their data for tylose gel spheres. An average absolute prediction error of 3.29% was obtained by the method of Cleland and Earle [5] when used to predict freezing times from their data set. The average absolute prediction error of the Cleland and Earle method when compared to all spherical food data was 9.92% with a 95% confidence range of ±1.99%.

Both of the methods developed by Pham [10,11] accurately predicted the freezing times of both the Cleland and Earle [5] data set and the Tocci and Mascheroni [20] data set, achieving an average absolute prediction error of less than 7.06%. Comparison of both of Pham's methods with the Ilicali and Saglam [12] data set for apples, however, produced an average absolute prediction error of 34.3%. Overall, the average absolute prediction error was less than 12.3% with a 95% confidence range less than ±2.50% for both of Pham's methods.

TABLE 3
Statistical Analysis of Food Freezing Time Estimation Methods Applicable to Spheres

Estimation Method	Average Absolute Prediction Error (%)	Standard Deviation (%)	95% Confidence Range (%)	Kurtosis	Skewness
Cleland and Earle [5]	9.92	9.94	±1.99	2.24	1.60
Ilicali and Saglam [12]	6.85	4.75	±0.952	0.680	0.930
Lacroix and Castaigne [7-9]	11.0	12.4	±2.49	3.83	2.01
Pham [10]	12.3	12.5	±2.50	1.62	1.59
Pham [11]	10.4	12.4	±2.49	2.89	1.88
Salvadori and Mascheroni [17]	7.53	6.85	±1.37	3.25	1.72

The method of Ilicali and Saglam [12] yielded its best results when compared to their data set on apples. When compared to this data set, an average absolute prediction error of 6.39% was obtained. For all spherical data, Ilicali and Saglam's method produced the lowest average absolute prediction error, 6.85%, also the narrowest 95% confidence range, ±0.952%.

The method of Lacroix and Castaigne [7-9] performed similarly to that of Pham's [10,11] methods. Overall, the Lacroix and Castaigne method yielded an average absolute prediction error of 11.0% with a large 95% confidence range of ±2.49%. The method of Salvadori and Mascheroni [17] performed well overall, yielding an average absolute prediction error of 7.53% with a fairly narrow 95% confidence range of ±1.37%.

Conclusions

A review of selected semi-analytical/empirical food freezing time estimation methods for regularly shaped food items was given in this paper. In addition, the performance of each of the methods was evaluated by comparing their results to experimental freezing time data found in the literature.

The food freezing time estimation methods developed by Cleland and Earle [4,5] performed well for infinite slabs and infinite cylinders. The methods of Pham [10,11] performed better with infinite slabs and infinite cylinders of food than they did for spherical food items. The method of Ilicali and Saglam [12] produced low prediction errors for cylindrical and spherical food items, but produced large prediction errors for infinite slabs of food. The method of Lacroix and Castaigne [7-9] performed best for infinite cylinders of food and produced high prediction errors for infinite slabs and spheres. The method of Salvadori and Mascheroni [17] performed consistently with all three regular shapes, producing moderately large prediction errors.

In summary, for infinite slabs, the methods of Pham [10,11] Hung and Thompson [6] and Cleland

and Earle [4] all performed equally well. For infinite cylinders, the methods of Pham [11] and Cleland and Earle [5] performed the best while the methods of Ilicali and Saglam [12] and Lacroix and Castaigne [7-9] also did well. Finally, for spheres, the methods of Ilicali and Saglam [12] and Salvadori and Mascheroni [17] gave the best results.

Nomenclature

Bi	Biot number
C_l	volumetric specific heat of unfrozen food
C_s	volumetric specific heat of fully frozen food
D	slab thickness or cylinder/sphere diameter
f	cooling time parameter
f_1	cooling time parameter for precooling
f_3	cooling time parameter for subcooling
h	heat transfer coefficient
j	cooling time parameter
j_1	cooling time parameter for precooling
j_3	cooling time parameter for subcooling
k_s	thermal conductivity of fully frozen food
L_f	volumetric latent heat of fusion
P	Plank's geometry factor
Pk	Plank number; $C_l(T_i - T_f)/?H$
R	Plank's geometry factor
Ste	Stefan number; $C_s(T_f - T_m)/?H$
t	freezing time
t_1	precooling time
t_2	phase change time
t_3	subcooling time
T_c	final center temperature of food item
T_f	initial freezing temperature of food item
T_i	initial temperature of food item
T_m	freezing medium temperature
_	average temperature of food item
X	dimensionless freezing time parameter
$?H$	volumetric enthalpy difference
$?H_{10}$	volumetric enthalpy difference between the initial freezing temperature, T_f, and -10°C
$?H_{18}$	volumetric enthalpy difference between the initial temperature, T_i, and -18°C

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