Evaluation of Semi-Analytical/Empirical Freezing Time Estimation Methods Part II: Irregularly Shaped Food Items

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The freezing of food is one of the most significant applications of refrigeration. In order for freezing operations to be cost-effective, it is necessary to optimally design the refrigeration equipment. This requires estimation of the freezing times of foods. Part I quantitatively assessed selected semi-analytical/empirical food freezing time prediction methods that apply to regularly shaped food items. Part II covers techniques that apply to irregularly shaped food items. The performance of these various methods is quantitatively evaluated by comparing their numerical results to a comprehensive experimental freezing time data set compiled from the literature.

INTRODUCTION

This is the second part of a two-part paper that quantitatively evaluates selected semi-analytical/empirical food freezing time prediction methods. As discussed in Part I (Becker and Fricke 1998), this analysis was conducted to aid the designer of food freezing equipment who must estimate the freezing times of foods as well as the corresponding refrigeration loads.

Part I applies to regularly shaped food items, while this Part II focuses on techniques that apply to irregularly shaped foods. These techniques require a two-step procedure in which the freezing time is first estimated by using one of the methods applicable to regularly shaped food items, as described in Part I. For irregularly shaped food objects the Equivalent Heat Transfer Dimensionality method, first introduced by Cleland and Earle (1982); the Mean Conducting Path method developed by Pham (1985); and the Equivalent Sphere Diameter method developed by Ilicali and Engez (1990) and Ilicali and Hocalar (1990) are appropriate. The performance of each of these freezing time estimation methods applicable to irregular shapes is quantitatively evaluated by comparing numerical results to a comprehensive experimental freezing time data set compiled from the literature.

EQUIVALENT HEAT TRANSFER DIMENSIONALITY

Cleland and Earle (1982) introduced a geometric correction factor, called the “equivalent heat transfer dimensionality” E to calculate the freezing times of irregularly shaped food items. The freezing time of an irregularly shaped object t_{shape} is related to the freezing time of an infinite slab t_{slab} via E as follows:

$$t_{shape} = \frac{t_{slab}}{E} \tag{1}$$

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The freezing time of the infinite slab is calculated from one of the many suitable freezing time estimation methods available for infinite slabs, as discussed in Part I.

Using data collected from a large number of freezing experiments, Cleland and Earle (1982) developed empirical correlations for the equivalent heat transfer dimensionality applicable to rectangular bricks and finite cylinders. For rectangular brick shapes with dimensions $D \times \beta_1 D \times \beta_2 D$, the equivalent heat transfer dimensionality was given as follows:

$$E = 1 + W_1 + W_2 \tag{2}$$

where

$$W_1 = \left(\frac{\mathrm{Bi}}{\mathrm{Bi} + 2}\right)\frac{5}{8\beta_1^3} + \left(\frac{2}{\mathrm{Bi} + 2}\right)\frac{2}{\beta_1(\beta_1 + 1)} \tag{3}$$

and

$$W_2 = \left(\frac{\mathrm{Bi}}{\mathrm{Bi} + 2}\right)\frac{5}{8\beta_1^3} + \left(\frac{2}{\mathrm{Bi} + 2}\right)\frac{2}{\beta_2(\beta_2 + 1)} \tag{4}$$

In Equations (3) and (4), the dimensional ratios β_1 and β_2 are defined as:

$$\beta_1 = \frac{\text{second shortest dimension of the food item}}{\text{shortest dimension of the food item}} \tag{5}$$

$$\beta_2 = \frac{\text{longest dimension of the food item}}{\text{shortest dimension of the food item}} \tag{6}$$

and Bi is the Biot number, $\mathrm{Bi} = hD/k_s$, where h is the surface heat transfer coefficient, D is the shortest dimension of the food item, and k_s is the thermal conductivity of the fully frozen food item. For the case of finite cylinders where the diameter is smaller than the height, the equivalent heat transfer dimensionality was given as follows:

$$E = 2.0 + W_2 \tag{7}$$

In addition, Cleland et al. (1987a, 1987b) developed expressions for determining the equivalent heat transfer dimensionality of infinite slabs, infinite and finite cylinders, rectangular bricks, spheres, and two- and three-dimensional irregular shapes. Numerical methods were used to calculate the freezing or thawing times for these various shapes and a non-linear regression analysis of the resulting numerical data yielded the following form for the equivalent heat transfer dimensionality:

$$E = G_1 + G_2 E_1 + G_3 E_2 \tag{8}$$

where

$$E_1 = X\left(\frac{2.32}{\beta_1^{1.77}}\right)\frac{1}{\beta_1} + \left[1 - X\left(\frac{2.32}{\beta_1^{1.77}}\right)\right]\frac{0.73}{\beta_1^{2.50}} \tag{9}$$

$$E_2 = X\left(\frac{2.32}{\beta_2^{1.77}}\right)\frac{1}{\beta_2} + \left[1 - X\left(\frac{2.32}{\beta_2^{1.77}}\right)\right]\frac{0.73}{\beta_2^{2.50}} \tag{10}$$

$$X(x) = x/(\mathrm{Bi}^{1.34} + x) \tag{11}$$

where the geometric constants G_1, G_2, and G_3 are given in Table 1.

Table 1. Geometric Constants from Cleland et al. (1987a)

Shape	G_1	G_2	G_3
Infinite slab	1	0	0
Infinite cylinder	2	0	0
Sphere	3	0	0
Squat cylinder	1	2	0
Short cylinder	2	0	1
Infinite rod	1	1	0
Rectangular brick	1	1	1
Two-dimensional irregular shape	1	1	0
Three-dimensional irregular shape	1	1	1

Using the freezing time prediction methods for infinite slabs and various multi-dimensional shapes developed by McNabb et al. (1990), Hossain et al. (1992a) derived infinite series expressions for the equivalent heat transfer dimensionality of infinite rectangular rods, finite cylinders, and rectangular bricks. For most practical freezing situations, only the first term of these series expressions is significant. The resulting expressions for E are given in Table 2.

Hossain et al. (1992b) also presented a semi-analytically derived expression for the equivalent heat transfer dimensionality of two-dimensional, irregularly shaped food items. An equivalent “pseudo-elliptical” infinite cylinder was used to replace the actual two-dimensional, irregular shape in the calculations. A pseudo-ellipse is a shape which depends upon the Biot number. As the Biot number approaches infinity, the shape closely resembles an ellipse. As the Biot number approaches zero, the pseudo-elliptical infinite cylinder approaches an infinite rectangular rod. Hossain et al. (1992b) state that for practical Biot numbers, the pseudo-ellipse is very similar to a true ellipse. This model pseudo-elliptical infinite cylinder has the same volume per unit length and characteristic dimension as the actual food item. The resulting expression for E is given as follows:

$$E = 1 + \frac{1 + \frac{2}{\mathrm{Bi}}}{\beta^2 + \frac{2\beta}{\mathrm{Bi}}} \tag{12}$$

In Equation (12) the Biot number is based upon the shortest distance from the thermal center to the surface of the food item; not twice that distance, i.e., $\mathrm{Bi} = hD/(2k_s)$. Using this expression for E, the freezing time of two-dimensional, irregularly shaped food items t_{shape} can be calculated via Equation (1).

Hossain et al. (1992c) extended this analysis to the prediction of freezing times of three-dimensional, irregularly shaped food items. In this work, the irregularly shaped food item was replaced with a model ellipsoid shape having the same volume, characteristic dimension and smallest cross sectional area orthogonal to the characteristic dimension, as the actual food item. An expression was presented for the E of a pseudo-ellipsoid:

Table 2. Expressions for Equivalent Heat Transfer Dimensionally from Hossain et al. (1992a)

Shape	Expressions for Equivalent Heat Transfer Dimensionally, *E*
Infinite Rectangular Rod ($2L$ by $2\beta_1 L$)	$E = \left(1+\frac{2}{\mathrm{Bi}}\right)\left\{\left(1+\frac{2}{\mathrm{Bi}}\right) - 4\sum_{n=1}^{\infty}\left[\frac{(\sin z_n)}{z_n^3\left(1+\frac{\sin^2 z_n}{\mathrm{Bi}}\right)\left(\frac{z_n}{\mathrm{Bi}}\sinh(z_n\beta_1)+\cosh(z_n\beta_1)\right)}\right]\right\}^{-1}$ where z_n are the roots of $\mathrm{Bi} = z_n \tan(z_n)$ and $\mathrm{Bi} = hL/k$ where L is the shortest distance from the center of the rectangular rod to the surface.
Finite Cylinder, height exceeds diameter (radius L and height $2\beta_1 L$)	$E = \left(2+\frac{4}{\mathrm{Bi}}\right)$ $\left\{\left(1+\frac{2}{\mathrm{Bi}}\right) - 8\sum_{n=1}^{\infty}\left[y_n^3 J_1(y_n)\left(1+\frac{y_n^2}{\mathrm{Bi}^2}\right)\left(\cosh(\beta_1 y_n)+\frac{y_n}{\mathrm{Bi}}\sinh(\beta_1 y_n)\right)\right]^{-1}\right\}^{-1}$ where y_n are the roots of $y_n J_1(y_n) - \mathrm{Bi} J_0(y_n) = 0$; J_0 and J_1 are Bessel functions of the first kind, order zero and one, respectively; and $\mathrm{Bi} = hL/k$ where L is the radius of the cylinder.
Finite Cylinder, diameter exceeds height (radius $\beta_1 L$ and height $2L$)	$E = \left(1+\frac{2}{\mathrm{Bi}}\right)\left\{\left(1+\frac{2}{\mathrm{Bi}}\right) - 4\sum_{n=1}^{\infty}\frac{\sin z_n}{z_n^2[z_n + \cos z_n \sin z_n]\left[I_0(z_n\beta_1)+\frac{z_n}{\mathrm{Bi}}I_1(z_n\beta_1)\right]}\right\}^{-1}$ where z_n are the roots of $\mathrm{Bi} = z_n \tan(z_n)$, I_0 and I_1 are Bessel function of the second kind, order zero and one, respectively, and $\mathrm{Bi} = hL/k$ where L is the radius of the cylinder.
Rectangular Brick ($2L$ by $2\beta_1 L$ by $2\beta_2 L$)	$E = \left(1+\frac{2}{\mathrm{Bi}}\right)\left\{\left(1+\frac{2}{\mathrm{Bi}}\right) - 4\sum_{n=1}^{\infty}\left[\frac{\sin z_n}{z_n^3\left(1+\frac{\sin^2 z_n}{\mathrm{Bi}}\right)\left[\frac{z_n}{\mathrm{Bi}}\sinh(z_n\beta_1)+\cosh(z_n\beta_1)\right]}\right]\right.$ $- 8\beta_2^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\sin z_n \sin z_m\left[\left(\cosh(z_{nm})+\frac{z_{nm}}{\mathrm{Bi}\beta_2}\sinh(z_{nm})\right)\right.\right.$ $\left.\left.\left.z_n z_m z_{nm}^2\left(1+\frac{1}{\mathrm{Bi}}\sin^2 z_n\right)\left(1+\frac{1}{\mathrm{Bi}\beta_1}\sin^2 z_m\right)\right]^{-1}\right]\right\}^{-1}$ where z_n are the roots of $\mathrm{Bi} = z_n \tan(z_n)$, z_m are the roots of $\mathrm{Bi}\beta_1 = z_m \tan(z_m)$, and $\mathrm{Bi} = hL/k$ where L is the shortest distance from the thermal center of the rectangular brick to the surface, and z_{nm} is given as: $z_{nm}^2 = z_n^2\beta_2^2 + z_m^2\left(\frac{\beta_2}{\beta_1}\right)^2$

Table 3. Geometric Parameters from Lin et al. (1996b)

Shape	p_1	p_2	p_3
Infinite slab ($\beta_1 = \beta_2 = \infty$)	0	0	0
Infinite rectangular rod ($\beta_1 \geq 1$, $\beta_2 = \infty$)	0.75	0	−1
Brick ($\beta_1 \geq 1$, $\beta_2 \geq \beta_1$)	0.75	0.75	−1
Infinite cylinder ($\beta_1 = 1$, $\beta_2 = \infty$)	1.01	0	0
Infinite ellipse ($\beta_1 > 1$, $\beta_2 = \infty$)	1.01	0	1
Squat cylinder ($\beta = \beta_2$, $\beta_1 \geq 1$)	1.01	0.75	−1
Short cylinder ($\beta_1 = 1$, $\beta_2 \geq 1$)	1.01	0.75	−1
Sphere ($\beta_1 = \beta_2 = 1$)	1.01	1.24	0
Ellipsoid ($\beta_1 \geq 1$, $\beta_2 \geq \beta_1$)	1.01	1.24	1

$$E = 1 + \frac{1 + \frac{2}{\mathrm{Bi}}}{\beta_1^2 + \frac{2\beta_1}{\mathrm{Bi}}} + \frac{1 + \frac{2}{\mathrm{Bi}}}{\beta_2^2 + \frac{2\beta_2}{\mathrm{Bi}}} \tag{13}$$

In Equation (13), the Biot number is based on the shortest distance from the thermal center to the surface of the food item; not twice that distance, i.e., $\mathrm{Bi} = hD/(2k_s)$. With this expression for E, the freezing time of three-dimensional, irregularly shaped food items t_{shape} may be calculated using Equation (1).

The method of Lin et al. (1993, 1996a, 1996b) may also be used to determine equivalent heat transfer dimensionality for freezing time calculations. It is applicable to all of the geometric shapes given in Table 3. In the method of Lin et al. (1993, 1996a, 1996b), the equivalent heat transfer dimensionality, E, is given as a function of Biot number:

$$E = \frac{\mathrm{Bi}^{4/3} + 1.85}{\frac{\mathrm{Bi}^{4/3}}{E_\infty} + \frac{1.85}{E_o}} \tag{14}$$

Here, the Biot number is defined as $\mathrm{Bi} = hD/(2k_l)$ where k_l is the thermal conductivity of the unfrozen food item.

E_o and E_∞ are the equivalent heat transfer dimensionalities for the limiting cases of Bi = 0 and $\mathrm{Bi} \to \infty$, respectively. For two-dimensional, irregularly shaped food items, the equivalent heat transfer dimensionality for Bi = 0, E_o is given as:

$$E_o = \left(1 + \frac{1}{\beta_1}\right)\left[1 + \left(\frac{\beta_1 - 1}{2\beta_1 + 2}\right)^2\right] \tag{15}$$

For three-dimensional, irregularly shaped food items, E_o is given as:

$$E_o = \frac{2[\beta_1 + \beta_2 + \beta_1^2(1 + \beta_2) + \beta_2^2(1 + \beta_1)]}{2\beta_1\beta_2(1 + \beta_1 + \beta_2)} - \frac{[(\beta_1 - \beta_2)^2]^{0.4}}{15} \tag{16}$$

For finite cylinders, bricks, and infinite rectangular rods, E_o may be determined as follows:

$$E_o = 1 + \frac{1}{\beta_1} + \frac{1}{\beta_2} \tag{17}$$

For spheres, infinite cylinders and infinite slabs, $E_o = 3$, $E_o = 2$, and $E_o = 1$, respectively.

For both two-dimensional and three-dimensional food items, the general form for the equivalent heat transfer dimensionality at $\mathrm{Bi} \rightarrow \infty$, E_∞ is given as:

$$E_\infty = 0.75 + p_1 f(\beta_1) + p_2 f(\beta_2) \tag{18}$$

where

$$f(\beta) = \frac{1}{\beta^2} + 0.01 p_3 \exp\left[\beta - \frac{\beta^2}{6}\right] \tag{19}$$

with β_1 and β_2 as previously defined. The geometric parameters p_1, p_2, and p_3 are given in Table 3 for various geometries.

Table 4 summarizes the numerous methods which have been discussed for determining the equivalent heat transfer dimensionality of various geometries. These methods can be used in conjunction with Equation (1) to calculate freezing times of irregularly shaped food items.

Table 4. Summary of Methods for Determining Equivalent Heat Transfer Dimensionality

Slab		Cleland et al. (1987a,b) Equations (8) – (11)		Lin et al. (1996a,b) Equations (14) – (19)
Infinite cylinder		Cleland et al. (1987a,b) Equations (8) – (11)		Lin et al. (1996a,b) Equations (14) – (19)
Sphere		Cleland et al. (1987a,b) Equations (8) – (11)		Lin et al. (1996a,b) Equations (14) – (19)
Squat cylinder		Cleland et al. (1987a,b) Equations (8) – (11)	Hossain et al. (1992a) Table 8	Lin et al. (1996a,b) Equations (14) – (19)
Short cylinder	Cleland and Earle (1982) Equations (4),(7)	Cleland et al. (1987a,b) Equations (8) – (11)	Hossain et al. (1992a), Table 8	Lin et al. (1996a,b) Equations (14) – (19)
Infinite rod		Cleland et al. (1987a,b) Equations (8) – (11)	Hossain et al. (1992a), Table 8	Lin et al. (1996a,b) Equations (14) – (19)
Rectangular brick	Cleland and Earle (1982) Equations (2)–(4)	Cleland et al. (1987a,b) Equations (8) – (11)	Hossain et al. (1992a), Table 8	Lin et al. (1996a,b) Equations (14) – (19)
2-D Irregular shape (infinite ellipse)		Cleland et al. (1987a,b) Equations (8) – (11)	Hossain et al. (1992b), Equation (12)	Lin et al. (1996a,b) Equations (14) – (19)
3-D Irregular shape (ellipsoid)		Cleland et al. (1987a,b) Equations (8) – (11)	Hossain et al. (1992c), Equation (13)	Lin et al. (1996a,b) Equations (14) – (19)

MEAN CONDUCTING PATH

Pham's (1984, 1986) freezing time formulae, discussed in Part I, require knowledge of the food item's Biot number. To calculate the Biot number of a food item, its characteristic dimension must be known. Because it is difficult to determine the characteristic dimension of an irregularly shaped food item, Pham (1985) introduced the concept of the "mean conducting path." The mean conducting path $D_m/2$ is the mean heat transfer length from the surface of the food item to its thermal center. This path is used to calculate the Biot number, which in turn, is used in the calculation of the freezing time. Thus, the Biot number becomes:

$$\mathrm{Bi} = hD_m/k \tag{20}$$

where D_m is twice the mean conducting path.

For rectangular blocks of food, Pham (1985) found that the mean conducting path was proportional to the geometric mean of the block's two shorter dimensions. Based upon this result, Pham (1985) presented an equation to calculate the Biot number for rectangular blocks of food:

$$\frac{\mathrm{Bi}}{\mathrm{Bi}_o} = 1 + \left\{ [1.5\sqrt{\beta_1} - 1]^{-4} + \left[\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right)\left(1 + \frac{4}{\mathrm{Bi}_o}\right)\right]^{-4} \right\}^{-0.25} \tag{21}$$

where Bi_o is the Biot number based on the shortest dimension of the block D_1: $\mathrm{Bi}_o = hD_1/k$. The Biot number calculated with Equation (21) can then be substituted into the freezing time estimation methods of Pham (1984, 1986) to calculate the freezing time for rectangular blocks.

Pham (1985) noted that for squat shaped food items, the mean conducting path $D_m/2$ could be reasonably estimated as the arithmetic mean of the longest and shortest distances from the surface of the food item to its thermal center.

EQUIVALENT SPHERE DIAMETER

Ilicali and Hocalar (1990) and Ilicali and Engez (1990) introduced the "equivalent sphere diameter" concept to calculate freezing time of irregularly shaped food items. In this method, a sphere diameter is calculated that is based upon the volume and the volume to surface area ratio of the irregularly shaped food item. This equivalent sphere is then used to calculate the freezing time of the food item.

Considering an irregularly shaped food item where the shortest and longest distances from the surface to the thermal center were designated as D_1 and D_2, respectively, Ilicali and Hocalar (1990) and Ilicali and Engez (1990) defined the volume-surface diameter D_{vs} as the diameter of a sphere having the same volume to surface area ratio as the irregular shape:

$$D_{vs} = 6V/A_s \tag{22}$$

where V is the volume of the irregular shape and A_s is the surface area of the irregular shape. In addition, the volume diameter D_v was defined as the diameter of a sphere having the same volume as the irregular shape:

$$D_v = (6V/\pi)^{1/3} \tag{23}$$

Because a sphere is the solid geometry that has minimum surface area per unit volume, the equivalent sphere diameter $D_{eq,s}$ must be greater than D_{vs} and smaller than D_v. In addition, the contribution of the volume diameter D_v has to decrease as the ratio of the longest to the shortest dimensions, D_2/D_1, increases, since the object will be essentially two dimensional if $D_2/D_1 >> 1$. Therefore, the $D_{eq,s}$ was defined as follows:

$$D_{eq,s} = \frac{1}{\beta_2 + 1} D_v + \frac{\beta_2}{\beta_2 + 1} D_{vs} \tag{24}$$

Thus, the prediction of the freezing time of the irregularly shaped food item is reduced to predicting the freezing time of a spherical food item with diameter $D_{eq,s}$. Any of the freezing time estimation methods for spheres, discussed in Part I, may be used to calculate this freezing time.

Table 5. Empirical Freezing Time Data Set

Shape	Number of Data Points	Material	Reference
Squat cylinder	49	Scallop meat	Chung and Merritt (1991)
Squat cylinder	3	Beef	DeMichelis and Calvelo (1983)
Short cylinder	21	Mashed potato	Ilicali and Engez (1990)
Short cylinder	4	Ground beef	Ilicali and Engez (1990)
Short cylinder	6	Tylose gel	Hayakawa et al. (1983)
Rectangular brick	72	Tylose gel	Cleland and Earle (1979b)
Rectangular brick	17	Beef	DeMichelis and Calvelo (1983)
Rectangular brick	43	Mashed potato	Ilicali and Engez (1990)
Rectangular brick	9	Ground beef	Ilicali and Engez (1990)
2-D Irregular shape	42	Tylose gel	Cleland et al. (1987c)
2-D Irregular shape	4	Minced lean beef	Cleland et al. (1987c)
3-D Irregular shape	23	Beef	Ilicali and Hocalar (1990)
3-D Irregular shape	13	Tylose gel	Cleland et al. (1987c)

Table 6. Thermal Property Data Used for Calculation of Freezing Times

Property	Tylose Gel[a]	Mashed Potato[a]	Lean Beef[a]	Ground Beef[a]	Scallop Meat
k_l , W/(m·K)	0.55	0.53	0.50	0.44	0.53
k_s , W/(m·K)	1.65	1.90	1.55	1.45	1.87
C_l , μJ/(m^3·K)	3.71	3.66	3.65	3.38	3.90
C_s , μJ/(m^3·K)	1.90	1.95	1.90	1.95	2.15
L_f , μJ/m^3	209	235	209	188	268
T_f , °C	−0.6	−0.6	−1.0	−1.2	−2.2

[a] Data from Cleland and Earle (1984).

PERFORMANCE OF FOOD FREEZING TIME ESTIMATION METHODS

The performance of each of the previously discussed food freezing time estimation methods for irregularly shaped foods was analyzed by comparing calculated freezing times with empirical freezing time data available from the literature. The empirical freezing time data set is given in Table 5. In addition, the thermal properties of the food items used in this study are given in Table 6.

Tables 7 through 13 summarize the statistical analyses that were performed on the freezing time estimation methods discussed in this paper. For each of the methods, the following information is presented: the average absolute prediction error (%), the standard deviation (%), the 95% confidence range of the mean (%), the kurtosis, the skewness and the combined average absolute prediction error.

Performance of Equivalent Heat Transfer Dimensionality Methods

Because the equivalent heat transfer dimensionality methods rely on infinite slab freezing time estimation methods in order to predict the freezing time of irregularly shaped food items, several accurate freezing time prediction methods for infinite slabs were selected to evaluate the performance of the equivalent heat transfer dimensionality methods. These infinite slab freezing

Table 7. Performance of Equivalent Heat Transfer Dimensionality (EHTD) Methods for Squat Cylinders

EHTD Method	Freezing Time Method	Average Absolute Prediction Error, %	Standard Deviation, %	95% Confidence Interval for Mean, %	Kurtosis	Skew-ness	Prediction Error[a], %
Cleland and Earle (1982)	Cleland and Earle (1977)	30.0	26.5	±7.31	0.289	1.04	32.3
	Pham (1984)	31.4	25.4	±7.01	0.917	1.05	
	Pham (1986)	35.6	27.2	±7.49	−0.388	0.651	
Cleland et al. (1987a,b)	Cleland and Earle (1977)	42.7	24.0	±6.62	0.642	0.847	45.9
	Pham (1984)	45.2	22.0	±6.05	1.10	0.709	
	Pham (1986)	49.7	24.0	±6.61	0.426	0.484	
Hossain et al. (1992a)	Cleland and Earle (1977)	41.9	25.9	±7.14	0.455	0.906	45.0
	Pham (1984)	44.3	23.9	±6.59	1.02	0.869	
	Pham (1986)	48.9	25.9	±7.13	0.093	0.561	
Lin et al. (1996a,b)	Cleland and Earle (1977)	99.6	41.8	±11.5	0.703	1.02	104
	Pham (1984)	103	39.1	±10.8	1.78	1.18	
	Pham (1986)	110	41.1	±11.3	0.097	0.716	

[a] Combined average absolute prediction error for all three freezing time estimation methods

time estimation methods included those of Cleland and Earle (1977), Pham (1984) and Pham (1986). Thus, each equivalent heat transfer dimensionality method was tested three times, once with each of the three infinite slab freezing time estimation methods. The combined average absolute prediction error was then calculated as the average of these three test results.

Squat Cylinders. A squat cylinder is a finite cylinder whose diameter is greater than its height, as opposed to a short cylinder whose height is greater than its diameter. As shown in Table 7, all four equivalent heat transfer dimensionality methods exhibited difficulty in predicting the freezing times for the squat cylinder data sets of Chung and Merritt (1991) and DeMichelis and Calvelo (1983). These techniques produced large absolute prediction errors of 30% or more on average.

Short Cylinders. A short cylinder is a finite cylinder whose height is greater than its diameter, as opposed to a squat cylinder whose diameter is greater than its height. As shown in Table 8, the equivalent heat transfer dimensionality methods of Cleland and Earle (1982), Cleland et al. (1987a, 1987b) and Hossain et al. (1992a) all produced slightly better results for the short cylinder data sets than for the squat cylinder data sets. The method of Lin et al. (1996a, 1996b), however, performed poorly for the short cylinder data sets.

The combination of Pham's (1984) freezing time estimation method with the equivalent heat transfer dimensionality method of Cleland et al. (1987a, 1987b) performed the best for the short cylinder data sets, producing an average absolute prediction error of 19.3% with a standard deviation of 26.6%. The freezing time estimation method of Cleland and Earle (1977) in conjunction with the equivalent heat transfer dimensionality method of Hossain et al. (1992a) performed similarly, producing an average absolute prediction error of 19.4% with a standard deviation of 25.7% for the short cylinder data sets.

Bricks. As shown in Table 9, the equivalent heat transfer dimensionality methods developed by Cleland and Earle (1982) and Cleland et al. (1987a, 1987b) for brick shaped food items

Table 8. Performance of Equivalent Heat Transfer Dimensionality (EHTD) Methods for Short Cylinder

EHTD Method	Freezing Time Method	Average Absolute Prediction Error, %	Standard Deviation, %	95% Confidence Interval for Mean, %	Kurtosis	Skew-ness	Prediction Error[a], %
Cleland and Earle (1982)	Cleland and Earle (1977)	20.6	27.4	±10.1	2.63	1.90	21.3
	Pham (1984)	20.6	29.6	±10.9	2.36	1.85	
	Pham (1986)	22.8	34.0	±12.5	2.30	1.87	
Cleland et al. (1987a,b)	Cleland and Earle (1977)	19.6	24.4	±8.93	2.48	1.85	20.1
	Pham (1984)	19.3	26.6	±9.75	2.16	1.78	
	Pham (1986)	21.5	30.8	±11.3	2.14	1.82	
Hossain et al. (1992a)	Cleland and Earle (1977)	19.4	25.7	±9.44	2.34	1.84	20.6
	Pham (1984)	20.5	27.5	±10.1	2.05	1.77	
	Pham (1986)	22.0	32.1	±11.8	2.00	1.80	
Lin et al. (1996a,b)	Cleland and Earle (1977)	82.6	47.6	±17.4	1.90	1.56	90.6
	Pham (1984)	94.8	46.3	±17.0	1.63	1.42	
	Pham (1986)	94.3	54.0	±19.8	1.91	1.62	

[a] Combined average absolute prediction error for all three freezing time estimation methods

performed satisfactorily. The average absolute prediction error of the Cleland and Earle (1982) equivalent heat transfer dimensionality method for all three selected freezing time estimation methods combined was 7.88% with a standard deviation of no more than 8.65%. Similarly, for the Cleland et al. (1987a, 1987b) equivalent heat transfer dimensionality method, the combined average absolute prediction error was 6.89% with a standard deviation of no more than 7.72%. The combination of the Cleland and Earle (1977) freezing time estimation method with the equivalent heat transfer dimensionality method of Cleland et al. (1987a, 1987b) produced the lowest average absolute prediction error, 6.33%, for brick shaped food items.

The equivalent heat transfer dimensionality method of Hossain et al. (1992a) tended to underpredict freezing times with the three selected freezing time methods. The combined average absolute prediction error for the equivalent heat transfer dimensionality method of Hossain et al. was 7.03%, with a standard deviation of no more than 7.56%. The Hossain et al. (1992a) method exhibited relatively good consistency, as evidenced by the low values of standard deviation and high values of kurtosis. The Hossain et al. (1992a) equivalent heat transfer dimensionality method for bricks produced its best results in combination with Pham's (1984) freezing time estimation method, resulting in an average absolute prediction error of 6.77%.

For the three selected freezing time estimation methods, the Lin et al. (1996a, 1996b) equivalent heat transfer dimensionality method produced the largest combined average absolute prediction error of 9.76% with a standard deviation of no more that 8.42%. For brick shaped food items, the equivalent heat transfer dimensionality method of Lin et al. produced its best results with the freezing time estimation method of Pham (1984), resulting in an average absolute prediction error of 8.67% with a standard deviation of 7.17%.

2-D Irregular Shapes. As shown in Table 10, the equivalent heat transfer dimensionality method for 2-d irregular shapes developed by Hossain et al. (1992b) produced the lowest com-

Table 9. Performance of Equivalent Heat Transfer Dimensionality (EHTD) Methods for Rectangular Bricks

EHTD Method	Freezing Time Method	Average Absolute Prediction Error, %	Standard Deviation, %	95% Confidence Interval for Mean, %	Kurtosis	Skew-ness	Prediction Error[a], %
Cleland and Earle (1982)	Cleland and Earle (1977)	7.17	7.01	±1.17	26.9	3.78	7.88
	Pham (1984)	7.92	7.68	±1.28	12.0	2.71	
	Pham (1986)	8.55	8.65	±1.44	9.41	2.41	
Cleland et al. (1987a,b)	Cleland and Earle (1977)	6.33	6.67	±1.11	44.9	5.37	6.89
	Pham (1984)	7.25	7.10	±1.18	22.5	3.47	
	Pham (1986)	7.09	7.72	±1.29	21.0	3.47	
Hossain et al. (1992a)	Cleland and Earle (1977)	6.88	6.87	±1.14	28.8	4.83	7.03
	Pham (1984)	6.77	6.82	±1.14	28.5	4.03	
	Pham (1986)	7.44	7.56	±1.26	22.6	3.53	
Lin et al. (1996a,b)	Cleland and Earle (1977)	10.4	8.42	±1.40	15.2	2.60	9.76
	Pham (1984)	8.67	7.17	±1.19	25.0	3.52	
	Pham (1986)	10.2	8.22	±1.37	16.0	2.75	

[a] Combined average absolute prediction error for all three freezing time estimation methods

bined average absolute prediction error of 4.96%. This equivalent heat transfer dimensionality method performed well in conjunction with the freezing time estimation method of Pham (1986), producing an average absolute prediction error of 4.17% with a standard deviation of 3.49%. The freezing time estimation method of Cleland and Earle (1977) also performed well with this equivalent heat transfer dimensionality method, producing an average absolute prediction error of 4.37% with a standard deviation of 3.61%.

The equivalent heat transfer dimensionality method of Lin et al. (1996a, 1996b) produced satisfactory results for 2-d irregular shapes, with a combined average absolute prediction error of 9.16%. This equivalent heat transfer dimensionality method performed its best with the freezing time estimation method of Cleland and Earle (1977), producing an average absolute prediction error of 7.12% with a standard deviation of 5.16%.

The equivalent heat transfer dimensionality method developed by Cleland et al. (1987a, 1987b) produced relatively large absolute prediction errors and a combined average absolute prediction error of 12.6%. Its range of average absolute prediction errors was from 8.70% when using Cleland and Earle's (1977) freezing time prediction method to 13.2% when using Pham's (1986) freezing time prediction method.

3-D Irregular Shapes. Both the Hossain et al. (1992c) and the Lin et al. (1996a, 1996b) equivalent heat transfer dimensionality methods for 3-d irregularly shaped food items performed satisfactorily, as shown in Table 11. The Hossain et al. (1992c) method produced the lowest combined average absolute prediction error of 9.52% for the three selected freezing time estimation methods, with a standard deviation of no more than 7.96%. For 3-d irregularly shaped food items, the combined average absolute prediction error for the Lin et al. (1996a, 1996b) method was 10.5%, with a standard deviation of at most 8.36%. The combination of the freezing time estimation method of Cleland and Earle (1977) with the equivalent heat transfer dimensionality method of Hossain et al. produced the best results for the 3-d irregularly shaped food data set.

Table 10. Performance of Equivalent Heat Transfer Dimensionality (EHTD) Methods for 2-D Irregular Shapes

EHTD Method	Freezing Time Method	Average Absolute Prediction Error, %	Standard Deviation, %	95% Confidence Interval for Mean, %	Kurtosis	Skew-ness	Prediction Error[a], %
Cleland et al. (1987a,b)	Cleland and Earle (1977)	8.70	4.39	±1.30	−1.02	−0.002	12.6
	Pham (1984)	15.9	5.68	±1.69	−0.131	−0.585	
	Pham (1986)	13.2	4.57	±1.36	0.100	−0.368	
Hossain et al. (1992b)	Cleland and Earle (1977)	4.37	3.61	±1.07	4.71	1.88	4.96
	Pham (1984)	6.35	3.92	±1.16	−0.752	0.341	
	Pham (1986)	4.17	3.49	±1.04	0.040	0.990	
Lin et al. (1996a,b)	Cleland and Earle (1977)	7.12	5.16	±1.53	−0.185	0.676	9.16
	Pham (1984)	11.6	6.45	±1.92	−0.578	0.220	
	Pham (1986)	8.76	5.36	±1.49	−0.511	0.514	

[a] Combined average absolute prediction error for all three freezing time estimation methods

Table 11. Performance of Equivalent Heat Transfer Dimensionality (EHTD) Methods for 3-D Irregular Shapes

EHTD Method	Freezing Time Method	Average Absolute Prediction Error, %	Standard Deviation, %	95% Confidence Interval for Mean, %	Kurtosis	Skew-ness	Prediction Error[a], %
Cleland et al. (1987a,b)	Cleland and Earle (1977)	19.5	11.7	±3.97	−0.698	0.550	22.3
	Pham (1984)	28.4	14.5	±4.91	−0.680	0.242	
	Pham (1986)	19.1	11.4	±3.86	−0.776	0.638	
Hossain et al. (1992b)	Cleland and Earle (1977)	8.42	6.58	±2.23	−0.933	0.483	9.52
	Pham (1984)	11.3	7.96	±2.69	0.593	0.952	
	Pham (1986)	8.83	6.39	±2.16	−0.431	0.756	
Lin et al. (1996a,b)	Cleland and Earle (1977)	9.26	6.67	±2.26	−0.685	0.400	10.5
	Pham (1984)	12.9	8.36	±2.83	0.204	0.732	
	Pham (1986)	9.42	6.76	±2.29	−0.339	0.697	

[a] Combined average absolute prediction error for all three freezing time estimation methods

Table 12. Performance of the Mean Conducting Path Method for Rectangular Bricks

Freezing Time Method	Average Absolute Prediction Error, %	Standard Deviation, %	95% Confidence Interval for Mean, %	Kurtosis	Skew-ness	Prediction Error[a], %
Pham (1984)	7.83	7.19	±1.20	31.4	4.21	9.32
Pham (1986)	10.8	10.4	±1.72	8.45	2.06	

[a] Combined average absolute prediction error for both freezing time estimation methods

This combination resulted in an average absolute prediction error of 8.42% with a standard deviation of 6.58%.

The equivalent heat transfer dimensionality method of Cleland et al. (1987a, 1987b) did not perform well for the 3-d irregularly shaped food data set. The combined average absolute prediction error for this method was 22.3%.

Mean Conducting Path

Bricks. As shown in Table 12, the mean conducting path method for bricks (Pham 1985) in conjunction with the freezing time estimation method of Pham (1984) produced an average absolute prediction error of 7.83% with a standard deviation of 7.19% while its combination with Pham’s (1986) freezing time method produced a greater average absolute prediction error, 10.8%. Both combinations of Pham’s freezing time estimation methods with the mean conducting path method exhibited difficulty in predicting the freezing times of the beef data set from DeMichelis and Calvelo (1983) and the ground beef data set of Ilicali and Engez (1990), resulting in the relatively large average absolute prediction errors. The average absolute prediction errors for the two beef data sets ranged from 12% to 27%.

Equivalent Sphere Diameter

Because the equivalent sphere diameter method relies on spherical freezing time estimation methods in order to predict the freezing time of irregularly shaped food items, several accurate freezing time prediction methods for spheres were selected to evaluate the performance of the equivalent sphere diameter method. These spherical freezing time estimation methods included those of Cleland and Earle (1979a), Pham (1984) and Pham (1986). Thus, the equivalent sphere diameter technique was tested three times, once with each of the three spherical freezing time estimation methods. The combined average absolute prediction error was then calculated as the average of these three test results.

Squat Cylinders. As shown in Table 13, all three freezing time estimation methods in conjunction with the equivalent sphere diameter method produced large prediction errors for the squat cylinder data sets of Chung and Merritt (1990) and DeMichelis and Calvelo (1983). The Biot numbers for many of the experiments in the Chung and Merritt (1991) data set were very low ($Bi < 0.15$), and thus out of the range of the freezing time estimation method of Cleland and Earle (1979a). Hence, large prediction errors occurred, ranging from +60% to −1000%, when using the combination of the Cleland and Earle freezing time estimation method and the equivalent sphere diameter method. For the squat cylinder data set of Chung and Merritt (1991), Pham’s (1984, 1986) two freezing time estimation methods in conjunction with the equivalent sphere diameter method produced large prediction errors in the range of 5% to 135%.

Short Cylinders. As shown in Table 13, the equivalent sphere diameter technique, in conjunction with the three selected freezing time prediction methods, produced better results with the short cylinder data sets than with the squat cylinder data sets. The equivalent sphere diameter technique in combination with the freezing time estimation method of Cleland and Earle (1979a) produced the lowest average absolute prediction error, 8.81% with a standard deviation of 7.09%. The equivalent sphere diameter technique in conjunction with either of Pham's freezing time estimation methods exhibited great difficulty in predicting the freezing times of the tylose gel data set of Hayakawa et al. (1983). Ignoring this data set, the equivalent sphere diameter technique, in conjunction with both of Pham's freezing time estimation methods, yielded average absolute prediction errors of no more than 9.95%, with a standard deviation of less than 8.68%. The combined average absolute prediction error for short cylinders using the equivalent sphere diameter technique with the three selected spherical freezing time estimation methods was 18.4%.

Bricks. Overall, the equivalent sphere diameter technique combined with the freezing time estimation method of Pham (1984) produced good results with brick shaped food items. As shown in Table 13, this combination achieved an average absolute prediction error of 7.93% with a standard deviation of 7.41%. The combination of the equivalent sphere diameter technique with the Cleland and Earle (1979a) freezing time estimation method produced a large average absolute prediction error, 10.81%, and exhibited wide variation, as evidenced by the large standard deviation of 9.10%. The Cleland and Earle freezing time estimation method exhibited difficulty in predicting freezing times for the beef data set of DeMichelis and Calvelo (1983) due to Biot numbers which were out of the range for this correlation. Although the combination of the equivalent sphere diameter technique with Pham's (1984) freezing time estimation method produced a relatively large average absolute prediction error, 8.76%, this combination did exhibit greater consistency, as evidenced by the relatively low standard deviation of 7.41%.

The combined average absolute prediction error for bricks using the equivalent sphere diameter technique with the three selected spherical freezing time estimation methods was 9.16%.

2-D Irregular Shapes. As shown in Table 13, the equivalent sphere diameter technique with the three selected freezing time estimation methods did not perform well when compared to the 2-d irregular shape data set. These techniques produced large absolute prediction errors of 62% or more on average.

3-D Irregular Shapes. As shown in Table 13, the equivalent sphere diameter technique with the freezing time estimation method of Pham (1984) produced good results for 3-d irregularly shaped food items. This combination of methods achieved an average absolute prediction error of 8.65% with a standard deviation of 6.47%. The equivalent sphere diameter technique in conjunction with both the Cleland and Earle (1979a) and the Pham (1986) freezing time estimation methods performed similarly. These two combinations, on average, produced absolute prediction errors of no more than 11.7%.

The combined average absolute prediction error for 3-d irregularly shaped food items using the equivalent sphere diameter technique with the three selected spherical freezing time estimation methods was 10.5%.

CONCLUSIONS

A quantitative evaluation of selected food freezing time estimation methods for irregularly shaped food items was given in this paper, Part II. The performance of each method was quantified by comparing its numerical results to a comprehensive experimental freezing time data set compiled from the literature. The four equivalent heat transfer dimensionality methods applicable to finite cylinders were found to produce poor results when predicting the freezing times of squat and short cylindrically shaped food items.

The equivalent heat transfer dimensionality method of Cleland et al. (1987a, 1987b) in conjunction with the three selected freezing time estimation methods produced the best results for brick shaped food items. The equivalent heat transfer dimensionality method of Hossain et al. (1992a) also produced good results for brick shaped food items. For brick shapes, the Lin et al. (1996a, 1996b) method did not perform as well as the other three equivalent heat transfer dimensionality methods, but it did produce satisfactory results, yielding an average absolute prediction error of 9.76%.

For 2-d irregular shapes, the equivalent heat transfer dimensionality method of Hossain et al. (1992b) exhibited the lowest average absolute prediction error for the three selected freezing time estimation methods. In addition, the freezing time estimation method of Cleland and Earle (1977) produced good results with the equivalent heat transfer dimensionality method of Lin et al. (1996a, 1996b) for 2-d irregular shapes. Large absolute prediction errors were

Table 13. Performance of the Equivalent Sphere Diameter Method

Shape	Freezing Time Method	Average Absolute Prediction Error, %	Standard Deviation, %	95% Confidence Interval for Mean, %	Kurtosis	Skew-ness	Prediction Error[a], %
Squat Cylinder	Cleland and Earle (1979a)	188	263	±72.5	1.62	1.63	96.3
	Pham (1984)	47.9	25.5	±7.04	0.233	0.624	
	Pham (1986)	53.1	28.3	±7.80	–0.102	0.514	
Short Cylinder	Cleland and Earle (1979a)	8.81	7.09	±2.60	0.431	0.927	18.4
	Pham (1984)	22.8	29.9	±11.0	1.84	1.72	
	Pham (1986)	23.5	35.0	±12.8	1.79	1.75	
Rectangular Brick	Cleland and Earle (1979a)	10.8	9.10	±1.52	12.9	2.60	9.16
	Pham (1984)	7.93	7.41	±1.23	16.4	2.98	
	Pham (1986)	8.76	7.89	±1.31	15.6	2.81	
2-D Irregular Shape	Cleland and Earle (1979a)	64.9	14.5	±4.30	–0.593	–0.109	65.0
	Pham (1984)	67.9	15.7	±4.67	–0.256	–0.407	
	Pham (1986)	62.2	14.9	±4.43	–0.720	–0.323	
3-D Irregular Shape	Cleland and Earle (1979a)	11.1	7.51	±2.54	–1.20	0.336	10.5
	Pham (1984)	8.65	6.47	±2.19	–0.375	0.801	
	Pham (1986)	11.7	6.23	±2.11	–0.948	0.278	

[a] Combined average absolute prediction error for all three freezing time estimation methods

obtained with the use of the Cleland et al. (1987a, 1987b) equivalent heat transfer dimensionality method for 2-d irregular shapes.

For 3-d irregularly shaped food items, both the equivalent heat transfer dimensionality methods of Hossain et al. (1992c) and Lin et al. (1996a, 1996b) performed satisfactorily with the three selected freezing time estimation methods. The method of Cleland et al. (1987a, 1987b) exhibited large absolute prediction errors for 3-d irregularly shaped food items.

For rectangular brick shaped food items, the mean conducting path method of Pham (1985) produced its best results in conjunction with Pham's (1984) freezing time estimation method. The combination of the mean conducting path method and Pham's (1986) freezing time method produced slightly larger absolute prediction errors.

The equivalent sphere diameter technique performed satisfactorily in conjunction with the three selected freezing time estimation methods for rectangular brick shaped food items and 3-d irregularly shaped food items. However, the equivalent sphere diameter technique produced large absolute errors when predicting the freezing time of finite cylinders and 2-d irregularly shaped food items.

In summary, for short, cylindrically shaped food items, the equivalent heat transfer dimensionality methods of Cleland et al. (1987a, 1987b) and Hossain et al. (1992a) as well as the equivalent sphere diameter technique performed best. For rectangular bricks, the equivalent heat transfer dimensionality methods of Cleland and Earle (1982), Cleland et al. (1987a, 1987b) and Hossain et al. (1992a) performed well. For 2-d irregularly shaped food items, the equivalent heat transfer dimensionality method of Hossain et al. (1992b) performed the best.

The equivalent heat transfer dimensionality methods of Hossain et al. (1992c) and Lin et al. (1996a, 1996b) as well as the equivalent sphere diameter method exhibited good performance for 3-d irregularly shaped food items.

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NOMENCLATURE

A_s surface area of food item, m^2
Bi Biot number
Bi_o Biot number based on shortest dimension; $Bi_o = hD_1/k$
C_l volumetric specific heat of unfrozen food, $J/(m^3 \cdot K)$
C_s volumetric specific heat of fully frozen food, $J/(m^3 \cdot K)$
D characteristic dimension, m
D_1 shortest dimension, m
D_2 longest dimension, m
$D_{eq,s}$ equivalent sphere diameter, m
D_m twice the mean conducting path, m
D_v volume diameter, m
D_{vs} volume-surface diameter, m
E equivalent heat transfer dimensionality
E_o equivalent heat transfer dimensionality at Bi = 0
E_1 parameter given by Equation (9)
E_2 parameter given by Equation (10)
E_∞ equivalent heat transfer dimensionality at $Bi \rightarrow \infty$
G_1 geometric constant in Equation (8)
G_2 geometric constant in Equation (8)
G_3 geometric constant in Equation (8)
h heat transfer coefficient, $W/(m^2 \cdot K)$
$I_o(x)$ Bessel function of the second kind, order zero
$I_1(x)$ Bessel function of the second kind, order one
$J_o(x)$ Bessel function of the first kind, order zero
$J_1(x)$ Bessel function of the first kind, order one
k thermal conductivity of food item, $W/(m \cdot K)$
k_l thermal conductivity of unfrozen food, $W/(m \cdot K)$
k_s thermal conductivity of fully frozen food, $W/(m \cdot K)$
L half thickness of slab or radius of cylinder/sphere, m
L_f volumetric latent heat of fusion, J/m^3
p_1 geometric parameter from Lin et al. (1996b)
p_2 geometric parameter from Lin et al. (1996b)
p_3 geometric parameter from Lin et al. (1996b)
t_{shape} freezing time of an irregularly shaped food item, s
t_{slab} freezing time of an infinite slab shaped food item, s
T_f initial freezing temperature of food item, °C
V volume of food item, m^3
W_1 parameter given by Equation (3)
W_2 parameter given by Equation (4)
$X(x)$ function given by Equation (11)
y_n roots of transcendental equation; $y_n J_1(y_n) - \mathrm{Bi} J_o(y_n) = 0$
z_m roots of transcendental equation; $\mathrm{Bi}\beta_1 = z_m \tan(z_m)$
z_n roots of transcendental equation; $\mathrm{Bi} = z_n \tan(z_n)$
z_{nm} parameter given in Table 2
β_1 ratio of the second shortest dimension to the shortest dimension, Equation (5)
β_2 ratio of the longest dimension to the shortest dimension, Equation (6)

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